Modeling Causality

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ABSTRACT. This paper is a methodological exercise, applying model theory to causality. We start by explaining that model theory does not reduce to a formal tool, that it is an interesting and deep philosophical approach. We then develop a framework where causality is a binary relation between objects considered as events. From this perspective we examine the so-called principle of causality and we also discuss other possible axioms for the relation of causality analyzing their significance and import. We end by a case study: citation in research papers viewed as a cause-effect phenomenon

Dedicated to Patrick Suppes (1922-2014)
The Last Cowboy of Thought

$a \hookrightarrow b$

La cause la plus profonde se son malheur c'est qu'il pensait
qu'il n'y avait pas de fumée sans feu,
et quand la poudrière de sa voisine n'était pas allumée,
il avait peur que que tout s'envole en fumée.
Baron de Chambourcy

0 Toward a Theory of Causality

"Causality" is a word and there is an idea (notion, concept) corresponding to it. Furthermore this idea may apply to reality, it can describe and explain something in the world: the physical world, the biological world, the economical world, the psychological world, etc. This trinity word-idea-reality is not proper to causality. To the word "mud" corresponds a reality in which we can flounder, that in turn can be characterized by an idea, *earth mixed with water* (cf. Plato's *Theaetetus*). But we don't necessarily have a trilogy for any substantive, because the distinction between idea and reality is not always clear, think for example of infinity.

In this paper we intend to study the articulation of such kind of trinity using model theory taking as an example causality. This can be seen as a methodological exercise. But this is not just a game. We intend to reach understanding, both of causality and the methodology of model theory. We are using the expression "methodological exercise" to stress that our interest is not only for causality but also for the methodology, be it apply to causality or other notions.

One may claim that causality has no proper meaning. Such an argument may apply to Guilt, the G-spot or God. But who shall we blame for the lack of meaning? The word or the idea? It is difficult to blame a word. Words are used to express ideas. It is true that many words reflect the confusion of our thought. If we want to inquire what causality is, we certainly cannot stick to the word, developing a purely descriptive approach, the way the word (and its linguistic variations, "causalité", "causalidad", "causalità", "Kausalität", etc.) has been used from Hume to Judea Pearl [13].[1] The notion of causality, as many notions, is relatively independent of the word. Aristotle, famous for his theory of four causes, was using the Greek word "aitos" which has no linguistic relation with the neo-Latin word "cause". Despite this change of language and even if we do not agree with Aristotle's theory, there is no doubt that the target of these two words is the same: the cause of something is how and why this thing happens. In other cultures this notion has also been thought with different words. In Chinese for example, the word is "gù':[2]

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Taking in account the limited import of the word, one may want to develop a descriptive approach of the idea behind the word, ranging from Aristotle to

[1]In English "causality" and "causation" are both used. In some sense "causation" is better because, according to the Cambridge dictionary, it means "the process of causing something to happen or exist" by contrast with causality defined as "the principle that there is a cause for everything that happens", involving a confusion between causality and the principle of causality. The reason why however we are using "causality" is due to its variations in other European languages where there are no equivalent to "causation".

[2]Thanks to Zhenzhen Guo for this indication and references to the *Mozi*.

David Bohm [7], but this is also limited. What we can do is to develop a *theory* of causality, trying to see how we can think about causality in a systematic, clear, meaningful and useful way. Such a theory cannot be absurdly normative; in this case there would be no reason to still use the word "causality", it would be a theory of something else. We have to find a good balance between descriptivity and normativity. Tarski has developed his theory of truth in this spirit [18]. To do so we have to take in account the meanings which have been attached to the word "causality" and the notion of causality.

However we have to be aware that in exact sciences sometimes the theory of a notion is quite remote from its usual meaning. Consider for example the case of time. This is maybe because the very nature of time is quite different from the vulgar idea of time. The way to an advanced scientific theory of a notion is not necessarily a straightforward road leading us deeper and deeper, higher and higher. It can be a tortuous path driving us to an unexpected notion we were not able to dream of. But in this case the reason why we use the same word for the remote reached point and the starting point is because the starting point was the starting point.[3]

So perhaps by developing a theory of causality we may reach something quite different from the common notion, because the very nature of causality is remote from the basic idea of causality. Someone may not believe in the *very nature* of causality, or *very nature* of time. This can indeed be just a way of speaking. We can instead speak of a *more sophisticated* notion, *more useful* or/and *applying better to reality*.

There is a difference between on the one hand a notion like reasoning, and on the other hand a notion like star. In the case of star, there are stars in reality and we want to develop a theory of this notion that describes, fits, grasps, explains this reality. We can say the same about a phenomenon like fear. In the case of reasoning, we may want to develop a normative theory. Normative in the sense not only that it is a better way to think about reasoning different from the confused ordinary idea, but normative in the sense that this is how reasoning should be performed. There is in this case an interaction between the theory and reality. The theory is transforming reality. Reasoning is not an independent reality.

A notion like time is more like the notion of star or fear, although it is more complex. In the case of causality it is more ambiguous, because one may argue that causality is just a way to think about reality. However "a way to think" of reality can be considered as a reality not in a noumenal sense, but in a phenomenal sense, to use Kant's terminology. Saying it is a reality basically means here that it has a nature which is independent of what we can think about it. A philosopher like Schopenhauer has presented a phenomenal theory of causality in a neo-Kantian sense: causality for him is one of the four roots of the principle of sufficient reason [16], this principle being a phenomenal reality that we cannot transform, but that we can describe more or less adequately.

[3]We have however to be cautious not to commit a Columbus confusion: natives of America have been called "Indians" but they have nothing to do with India. Having in view an idea we may reach another one ...

One may argue that causality does not properly exist, that it is a primitive and pre-historical notion that has no sense nowadays, that the cartography of our world of ideas has so much changed that it is as absurd to talk of causality in the contemporary world as to talk of Prussia or Aether.[4] But one may also sustain that causality is a notion that can be transformed, that we can develop a normative theory of causality that makes sense. Trying to develop a theory of causality is a way to see if past and future theories of causality are possible or not, if this notion has ever made sense or will ever make sense. Recently this notion has been discussed quite a lot, mainly by philosophers (see e.g. [1]), perhaps because it is a general notion that at this stage only philosophers can think about and hopefully save. Thinking about causality is a challenge for philosophy, but we have to see if it is possible to develop intelligent philosophy going beyond wordy discussions and disputes.

Our work here about causality is an exercise in philosophy. We will see how it is possible or not to develop a theory of causality using model theory.[5] We don't think that philosophy has necessarily to be developed using model theory, or that philosophy consists of developing theories. But what we want to show is that by so doing we may clarify the notion of causality.

1 The general framework: causality-effects as a binary structure

1.1 The methodology of model theory and causality

Let see how we can think about causality. Our idea here is to use model theory in a simple but authentic way. Model theory is a meta-theory in the sense that it is a theory to develop theories. Model theory can be applied to mathematics: theories of numbers, of spaces, of groups, fields but also to any non-mathematical field ... It is true that model theory has been mainly developed for mathematical theories and for this reason is strongly associated with mathematics. But the originator of model theory, Alfred Tarski,[6] had a

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[4]One century ago Bertrand Russell wrote: "In the following paper I wish, first, to maintain that the word "cause" is so inextricably bound up with misleading associations as to make its complete extrusion from the philosophical vocabulary desirable." [15]

[5]Our paper is written in such a way that it can be understood by someone who knows nearly nothing about model theory. But at the same time it can be of interest for model-theoretists. We don't present here a simplification of model theory that could be boring for the specialist, but rather a view of it, not too technical, stressing its philosophical value. Let us also emphasize that it is possible to find in the literature some approaches of causation quite close to the framework we are presenting, events related by a binary relation of causality, see e.g. [11]. Maybe in these approaches the authors are using "informally" model theory, but our aim here is to explicitly use model theory.

[6]Someone may argue that model theory was existing before Tarski invented the expression "model theory". But here there is a strong relation between the expression and the topic. Maybe it would be better to say that before Tarski coined the expression and develop the corresponding theory, this was just the pre-history of model theory. Hodges, in his seminal paper "Truth in a structure" [10], tries to explain why it took so-long to give the now standard first-order definition of truth.

general perspective and had interest to develop this theory for sciences like physics and biology. With Patrick Suppes and Leon Henkin he organized a conference on *Axiomatic Method* in December 1957 at Berkeley. This was a preliminary step for the launching of the series of congress *Logic, Methodology and Philosophy of Science* (LMPS), the first LMPS being organized by Suppes at Stanford University in 1960. Today model theory is not an important part of LMPS and of philosophy of science in general. What has become increasingly popular is "modelization", the exact meaning of this word not being very clear (see [12]). We want to show here how model theory can fruitfully be used, and can be seen as a good way to modelize.

The other reason why model theory is associated with mathematics is that it is rather a mathematical theory, although this point is not completely clear: it appears more mathematical for someone outside of mathematics than for a mathematician. The truth is that it is a very abstract theory that would be better classified as philosophical or metaphysical.[7] The basis of model theory is a structural point of view according to which we have objects and relations between objects. Some objects given together with some relations form a *structure*.[8] Here objects are conceived in a very general sense. They are not necessarily objects like a stone or a number, it can be an emotion, rain, any phenomenon. What is important in model theory is the two level perspective: objects on the one hand, relations on the other hand, objects being understood through the properties of the relations between objects.[9] These properties are characterized by some axioms.

A group of axioms is called *a theory*.[10] A theory has different *models*, structures obeying the axioms, structures which can be more or less different.[11] Sometimes one may want to reach unicity, i.e. all models are the same, in this case the theory is said to be *categorical*. This option is important in particular when one has in view the objects of the structure, for example if one wants to characterize natural numbers. If a theory of natural numbers has very different models this means that we are not succeeding to characterize what is a natural number. This is in fact exactly what happens: in the theory called "Peano arithmetics" we have non-standard models with strange objects. This is an important result due to Tarski himself.

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[7]The same can be said about first-order logic, model theory being one fundamental aspect of first-order logic.

[8]The notion of structure can be considered as a primitive notion and a set can be considered as a limit case of structure.

[9]These objects can be anything; they can also therefore be relations. But at this stage they are considered as objects by contrast to the relations which are considered to study them in the structure. In model theory there can also be functions, but we will not discuss this notion here. Functions can be considered as a particular type of relations.

[10]Here "theory" has not the same meaning as in "model theory", or in "number theory", a detailed analysis of the words "theory" and "model" can be found in [5].

[11]In model theory, we have on the one hand "theories", on the other hand "structures". These structures can in general be called "models", considering that a structure can always be a model of some theory.

If one has in view relations rather than objects, it is not necessarily important to look for categoricity. For example the notion of order is characterized by the axioms of antisymmetry and transitivity and it has many different models describing all the variations of this notion that we don't want to eliminate. This is not incompatible with the fact that one may want to focus on a particular notion of order and to categorically catch it, for example the notion of dense order. The notion of density can be easily captured because it is easy to find an axiom for density; nevertheless this axiom is not enough to get categoricity. To have a categorical theory of dense order we must choose between secondary features, in particular having or not a first or a last element. For this reason there are different categorical theories of dense order, the most famous being the one having for models exactly the rational numbers. These theories are incompatible in the sense that they don't have common models.

Developing model theory for causality we have to be aware of this distinction. The situation of causality can perhaps be compared to dense order in the sense that we may want to categorically characterize causality, to catch the very idea of causality. But the model theoretical perspective clearly shows us that we can have a less categorical approach, having like for order relations a central set of axioms extending in many different specific theories of causality incompatible with each other, which is not necessarily a problem. A relation of order can be discrete or dense, these are incompatible properties, nevertheless these two theories have a common ground, the two basic axioms for order.

The fact that we have different models of the same theory of causality may be seen as the variations of the notion of causality according to various "interpretations". From a structuralist point of view, if two structures are different, this means that the objects of the structures are different. Different models of the theory of causality can therefore be seen as different fields to which the notion of causality applies.

1.2 The relation of causality in a model-theoretical perspective

The simplest non trivial situation in model theory is when we have only one kind of objects and one relation between these objects, a binary relation. It is possible to show that more or less artificially any structure can be reduced to such kind of simple structure. This kind or reductionism is a technical result that can be useful for general metatheorems, but not necessarily interesting if we want to be close to the concepts we want to characterize. It happens that we can think about causality quite naturally using such a simple structure.

We consider structures with a binary relation on a set of objects, that we call *relation of causality* or *causality* for short and that we can symbolically represent by the capital letter "C". Given two objects a and b, we write aCb and this can be read as "a is in causal relation with b", or more simply "a causes b". In the second formulation we use a verb, this is not a problem at all, model theory is much more general that people usually think having in mind some specific applications of it. The notion of relation is something very abstract, and can be an (inter)action between two objects and many other "things".

When we use the letter "C", this can be viewed in two ways: an abbreviation and an abstraction. These two ways are not opposed but they also do not reduce one to the other. It is in fact not just an abbreviation, for this reason, and to avoid this ambiguity it is better to use another sign which is also more suggestive and symbolic like "$\hookrightarrow$". Then we write $a \hookrightarrow b$, expression we read as "a causes b".

This is quite similar as what is done with the relation of order, when we write $a < b$. Note that, except the rather complicated expression "a is in order relation with b" (which does mot clearly appear as different to "b is in order relation with a" - an unfortunate ambiguity especially for the case of an antisymmetric relation), there is no direct reading of this expression, because expressions like "a is inferior to b" or "a precedes b" are already interpretations of the relation of order.

Another example of relation is the relation of consequence. We have the expression $a \vdash b$ which is read as "b is a consequence of a a". The relation between the two objects is a relation of *consequence* in the same way that in the other cases we have relations of *causality* and *relation of order*. But then the word "consequence" is transposed into the object which is at the right of the relation, which is qualified as a "consequence".

In the case of causality the word naming the relation can be transposed on the left, to qualify the object at the left: "a is a cause of b". In the case of the relation of order, the word naming the relation is transposed neither on the left nor on the right. In the case of causality, we have also a name for the object which is at the right, it is called an *effect*. When $a \hookrightarrow b$ we say that b is an effect of a. In the case of a consequence relation, we can have also a word to qualify the other side of the relation, $a \vdash b$, we can say that a is a hypothesis (for b).

Like in the case of consequence there is a kind of disparity since the name of the relation is transposed only on one side. But what is important is that from our model theoretical perspective the objects of both sides of the relation of causality are of the same nature. This homogeneity corresponds to a framework not suitable for Aristotle's theory of causality or other theories. In the case of Aristotle's theory, on the one hand it would be difficult to argue that the four different causes are of the same nature and on the other hand that the effect, product of these four causes is of the same nature as these causes.

The fact that we use different names can be interpreted as meaning that these objects may have two different roles corresponding to their "positions". This way of speaking is usual in natural language for various situations: someone may be at the same time a son and a father. In our ordinary language it is not however clear that the words "cause" and "effect" just correspond to roles, maybe the idea is that they also correspond to two different kinds of objects. This is not really clear and in fact it is not clear also what kinds of objects correspond to either of these words. The word "object" would not even be used, there are no proper names for the things corresponding to these words. The way of naming here is a kind of reduction of something to its function. We will try here to construct objects behind the words, through a structural approach, which is not a pure functionalism.

1.3 Events

In model theory, given a structure, it is quite normal to speak of "objects" or "elements" of the structure. The word object is used in an abstract sense not corresponding to the common use of the word and when were are studying a particular class of models, we give to these objects the name of intended interpretations of these objects.

Here we will choose to call the objects "events". Let us keep emphasizing that since we are making a theory, the word "event" and correlated notions ("cause", "effect", "causality relation") are not considered in a purely descriptive sense. But we are trying to take a word which is as neutral as possible, and we are not dealing only with the word "event" but also with the notion associated to it.

Other words/notions are possible like "phenomenon" or "process", the idea being to choose moving things rather than static things. It makes sense to say that the work of the carpenter is the cause of the table. But since the idea of our model-theoretical framework is to have on both sides of the relation of causality the same kind of objects, we can instead say that the work of the carpenter is the cause of the production of the table, production of the table being an event, and the table a byproduct that we assimilate with this event.

Event can be seen as a quite general notion. Physical atomism reduces everything to atoms considered as small indivisible material objects. In the case of logical atomism, we have atomic propositions, corresponding to facts, which cannot be decomposed into other propositions, corresponding to more elementary facts. One may want to defend an "event atomism" considering that the world is based on some atomic events that cannot be divided. The option is not incompatible with our present theory, this is a particular case of it we are not especially defending here, but we leave this option open.

To further develop such a theory it would be useful to introduce in our structure some functions composing events into other events. According to our simple structure, the events are not necessarily considered as indecomposable, and also we do not suppose that there are such indecomposable events. We defend here a structuralist approach: what is important are the relations between events, and these relations are understood through a single notion, the notion of causality.

Let us emphasize that our perspective is not absolutist: though this structure causality-events is very general, it is only a way to look at the world. We can say that our theory causality-events is *universal*, and it is an important feature of it, in the sense that it encompasses all phenomena, but it is *relative* in the sense that it is just one possible way to look at them.

The standard meaning of "event" is: " anything that happens ". This sounds very general and it makes sense to say that anything happens, so that anything can be considered as an event. One may consider that an event is an interaction between some "things", like a stone thrown at a window. Here we have two "things": stone and window. But in our present theory we will not decompose events in more elementary things. Putting events in the first place means that we are considering what is happening and only that, in the same way that in propositional logic one does not decompose the proposition in other kinds of entities.

One may consider than the relation of causality is of a similar type as the relation of consequence rather than the relation of order, for a reason of multiplicity. Although we have talked about the consequence relation through an example such as $a \vdash b$, generally we have something like $a_1, ..., a_n \vdash b$, i.e. from various hypotheses we reach a conclusion. For causality we can also naturally consider that we have a multiplicity of causes leading to an effect: $a_1, ..., a_n \hookrightarrow b$. But we can consider that the multiple causes $a_1, ..., a_n$ together correspond to a single event. Of course it would be useful to have a function combining multiplicity of events into one single event, like we have connectives combining a multiplicity of propositions in one single proposition. But we can work first at a more abstract level without such functions. This analysis can also be applied on the right of the causality relation, we can consider, like in the case of multi-conclusion consequence relation, that there are many objects on the right, a cause having several effects. These several effects can be considered as one effect, "product" of these different effects, following the above analyzis for cause.

2 The principle of causality

2.1 One fundamental principle

Many questions that have been discussed about causality can quite easily be reformulated in our framework. We can analyze the new dimension that it shed on them but we also always have to be careful about the specificity and limitation of this perspective.

There is the question to know if everything has a cause. This can be understood in various ways. In our approach, this proposition can be stated as "Every event is caused by another event", which corresponds to the following first-order formula: $\forall x \exists y (y \hookrightarrow x)$. This can be considered as a formulation of *the principle of causality*.[12]

This principle is quite mythological, mythology promoted by a lot of ambiguities. The word "principle" itself is rather ambiguous. The word "axiom" is more neutral and relative, especially in the context of modern axiomatic and model theory.

There exists an ambiguity that persists even if we use the expression *the axiom of causality* instead of the expression *the principle of causality*: it suggests that this is the "one fundamental" thing that describes or/and defines causality. Imagine that we were using the expression "the axiom of order" to designate the axiom of antisymmetry. This is not the general way of speaking because the idea is that the notion of order is defined at least with two different axioms: antisymmetry and transitivity. Nevertheless one could argue in favor of using the expression "the axiom of order" for antisymmetry defending a position according to which, though antisymmetry is not the only axiom for order, it is the most fundamental one. Such expression does not necessarily imply pure uniqueness.

[12] In our framework "Every event is caused by another event" is equivalent to "Every event is an effect".

To find one axiom for causality, this is the least we can aim at. If we find several that can also be fine. But then we have to classify the different axioms: to see if there are some more fundamental than the other ones. In the case of the theory of order, we have two fundamental axioms – antisymmetry and transitivity – and then a bunch of “superficial” axioms, that extend the theory in different ways, such as the density axiom, the first element axiom, etc. To call the formula $\forall x \exists y (y \hookrightarrow x)$, “THE principle of causality”, means that this is the only one fundamental axiom for the theory of causality. This is a position than can be defended but one has to be aware of its significance.

2.2 The principle of causality and the principle of sufficient reason

“Everything has a cause” can be understood in different ways, depending on what kinds of things we are talking about: the cause of an economical crisis, the cause of a war, the cause of a storm, the cause of a illness, the cause of a depression.

There is a connection between the so-called *principle of causality* and *the principle of sufficient reason* (PSR) which is expressed as “Nihil est sine ratione”. Translating it into English and putting it in a positive form we have: “Everything has a reason”. This can be seen as a more general principle. This perspective has been precisely defended by Arthur Schopenhauer in his PhD *The fourfold root of the principle of sufficient reason*, where causality appears as one of the fourth forms of this principle, the one having to do with physical objects. Causality is one reasons among others, related to a specific field of objects. The theory of Schopenhauer is quite interesting. Without following necessarily the same division he is operating, one may want to argue that causality applies only to a certain class of objects.

In a sense this is what we are doing, because for us causality does apply only to events, not to any kind of things. But events are not really for us a part of reality, they rather are a partial way to look at reality. So our theory is not against applying causality to mental or psychic phenomena as is Schopenhauer’s theory (another form of the PSR different from the principle causality applies to mental phenomena according to the Danziger). Our approach is also not radically against applying causality to propositions / reasoning, saying that some hypotheses are causes of a theorem (for Schopenhauer this is is again another form of the PSR, different from causality).

When we are looking at axioms for causality, we don’t necessarily mean that mental phenomena, reasoning, physical objects are of the same type, we mean that viewed from a certain perspective, i.e. viewed as events, they obey some common axioms. Nowadays it is quite common to speak about “mental causation”. It is not clear that people are aware of the confusion that can result from this expression, assimilating mental phenomena with physical phenomena. Such an assimilation can be defended by physicalists, but we can talk about causality between mental events or between mental events and physical events without being a physicalist or a dualist (cf. the mind-body problem), with no commitment about the ontological nature of events.

2.3 The variety of models of the axiom of causality

To defend the axiom of causality understood as "Every event is caused by another event", one must be able to reject any counter-example, event without a cause, and/or be conscious of what is exactly implied or not by this statement, both from a philosophical and logical point of view,

That every event has a cause does not necessarily mean that there is a first cause, an event that is the cause of everything including itself, that can be called God, if we are kind enough to consider God as an event. This is an option among others. The principle of causality by itself is therefore neutral regarding such a God.

There are different models of the theory constituted by this sole axiom: one possible model is a model of infinite chains of events, another possible model is a model with cycles, we can also have a model which is a mix of the two. If one wants to eliminate these models, he has to put some additional axioms. And philosophically speaking one has to argue for these axioms. If one does not put these axioms, one has to be ready to explain the meaning of the variety of models of the axiom of causality. What we see is that one may have different reasons to defend the axiom of causality.

When claiming that everything has a cause, one may want to exclude multiplicity of disparate causes, even admitting that in general something may have different causes. Our framework here permits to defend this idea, because according to our formulation, every event is caused (at least) by a single event. As we have explained, this event can be viewed as a conjunction of events, but not any conjunction of events can necessarily be considered as a single event.

2.4 Rejection of the axiom of causality and counter-axiom

Let us see now if we can find some good reasons to reject the principle of causality. Rejecting this principle means that there is (at least) an event which has no cause, symbolically speaking: $\exists x \forall y (y \not\rightarrowtail x)$. Even if we have a good example / specimen to sustain this idea, we can wonder if it would make sense to take this formula as an axiom for a theory of causality.

This axiom, which is the negation of the principle of causality, starts with an existential quantifier and moreover includes a negation. Such configuration does not prohibit this formula to be an axiom: in famous theories we have a similar axiom. In a relation of total order $\exists x \forall y (y \not< x)$ means that there is a first element. But this would be strange to have only existential axioms, in particular for a theory describing reality, because the spirit of such "empirical" theory is to describe some general features of reality, and generality means universal quantification in the first place. Suppes and Chuaqui have shown that classical physics can be axiomatized by a theory only with universal quantifiers (and without negation), see [8].[13]

If we are not interested to have a complete and categorical theory we may

[13]Having in mind that causality and events are (a certain aspect of) reality, physical or mental, it makes sense to talk of the *law of causality* alternatively to the principle of causality.

choose neither $\forall x \exists y(y \hookrightarrow x)$ nor $\exists x \forall y(y \not\hookrightarrow x)$ as axioms. In this case we have models of the theory of causality in which every event has a cause, and others in which there are events without a cause. This not necessarily a problem in the sense that these two classes of models can be seen as describing two kinds of universe of events.

Let us see examine now other axioms that can be considered for the theory of causality.

3 Other axioms for causality

3.1 Everything has an effect

Everything has an effect can be formulated in our framework as "Every event causes another event", equivalent to "Every event is a cause". This appears as the reverse of the principle of causality and can be expressed by the following first-order formula: $\forall x \exists y(y \hookleftarrow x)$. Traditionally this appears as weaker than the principle of causality according to which if something happens there must be a reason, this is no purely contingent. This reverse principle does not even has a name! Like a stray dog ... But everybody understands its meaning, it is not like an unknown creature that we would encounter by surprise.

One of the reasons to deny this principle would be on the basis of identifying tininess with nothingness. But one has to be careful of the butterfly effect. Another reason is to stress the differences among phenomena, for example between thought and action. To kiss someone is not the same as thinking of kissing. Thinking may lead to action or ... nothing.

One can defend the reverse principle of causality in the perspective of a "relational" philosophy: not that everything is related to everything, but something is necessarily related to something else, upstream (as a cause) or downstream (as an effect). However a weak relationalism can be based on a disjunctive axiom, every event has a cause or an effect: $\forall x(\exists y(y \hookrightarrow x) \vee \exists y(y \hookleftarrow x))$.

3.2 Reflexivity

Does it make sense to say that an event is its proper cause? If one wants to rule out this possibility, nothing easier, she just has to choose the axiom: $\forall x(x \not\hookrightarrow x)$.

For a relation of order it is also possible to choose this option, in this case we speak of a *strict order*. It is also possible to choose reflexivity: everything is *superior or equal* to itself. In the case of a relation of order, it seems that one has to choose between these two options, a choice between two relations expressed by two different signs $<$ or $\leq$. The choice of the notation expresses the choice of one option. There is no notation for a relation of order with some elements which can be in relation with themselves, and others not.

In the case of a causal relation, it makes sense to have such kind of mix models. One may think that there is an event, which is cause of itself, and that this event is unique, a God in monotheism. To think that all events are causes of themselves, would be an exaggeration. But one may sustain that there is a class of events which are causes of themselves. A good way to develop this idea is to consider events which are causes of themselves but having no other causes than themselves: $\forall x((x \hookrightarrow x) \rightarrow (\forall y(y \hookrightarrow x) \rightarrow y = x)))$ These events can be interpreted as simulating events without causes and can permit to defend the axiom of causality in an original way.

Such events are causes of themselves and they are also effects of themselves, but there are no reason to consider that they don't have other effects than themselves. An event which is cause of itself and nothing else and which is at the same time effect of itself and nothing else would be a quite strange isolated phenomenon. On the other hand we can consider that there are events which have different causes but no effect, some kind of "dead-end" events. This makes sense in the case of generation and family.

3.3 Antisymmetry

If we consider that some events may be causes of themselves, then antisymmetry has to be formulated as follows: $\forall x \forall y((x \hookrightarrow y) \wedge (x \hookleftarrow y) \rightarrow x = y)$. Otherwise we can formulate it as: $\forall x \forall y((x \hookrightarrow y) \rightarrow (x \not\hookleftarrow y))$.

Many people would argue that causality is necessarily antisymmetric, mainly because they think that time is antisymmetric and that events are dependent of time. First let us point out that it is possible to have a theory of time which is not antisymmetric, having some circles or cycles - cf. Gödel's model of Einstein's theory [9].

On the other hand consider the following cycle of events: Spring leads to summer, summer leads to fall, fall leads to winter and winter leads back to spring. One may argue that we started with spring of 2014 and end up with spring of 2015, which is not the same spring. But we can considered a season independently of a particular instantiation of it. In the same way we can say that chicken cause eggs and eggs cause chicken, this a symmetric causality.

For the seasons one may argue that even if spring leads to fall (through summer) and fall leads to spring (through winter), a season is not an effect of a previous season, but that it "follows" the previous season. Nevertheless there are indeed many good reasons to see a more intrinsic relation between seasons. We must be cautious with argument such as "This is not causality because this is not antisymmetric". This is a kind of vicious circle argument, presupposing what we want to infer. A better argument against the above seasonable example would be to defend that causality is not transitive: spring causes summer and summer causes fall, but springer does not cause fall. However this non-transitive argument does not apply to chicken and eggs: there is nothing in-between.

Another way to defend symmetry of causality, even with antisymmetry of time, are interactive events, which are simultaneous in a certain portion of time. This is illustrated by our case study (see next section): a research paper can cause another one and vice-versa, there is a symmetric causation by simultaneous quotations.

3.4 Transitivity, Chain and Connectivity

Transitivity if clearly not an axiom for causality, although there may be some classes of events to which it applies. Let us emphasize than if we claim that an event *a* causes an event *b* and that an event *b* causes an event *c*, we can claim that *a* causes *c* even if the intermediate event *b* necessarily has to happen in-between. It makes sense if it is a chain of homogeneous events without outsiders, like the fall of dominoes.

Now consider that with your credit card you go to an ATM machine, take some cash and with this cash money buy some bananas in a street market where cash is not accepted. In this case we can say that your credit card is not the cause of buying these bananas, because it would not have been possible to directly buy bananas with your card. You credit card could not directly causes this effect. We can say that it is an "indirect" cause. When having a causality chain, we can distinguish between direct and indirect causes. An indirect cause is a cause such that there is necessarily an event in-between. But we have to be careful to make the distinction between dominoes and bananas.

The formal definition of chain for causality is the the same as for relation of order, even if we don't have transitivity. The concept of chain does not depend on transitivity. In fact the intutive idea of a chain is without transitivity.

We say that there is a chain between an event a and an event z, if there is a collection of events between a and z related by the relation of causality: $a \hookrightarrow b$, $b \hookrightarrow c$, ..., $y \hookrightarrow z$. And we adopt the following notation $a \lhook\joinrel\longrightarrow z$.

One may want to defend the idea that everything is connected, in the sense that given two events a and b, there is a chain of causality leading from a to b or from b to a. This can be formulated with the following axiom:

$\forall x \forall x ((x \lhook\joinrel\longrightarrow y) \vee (x \longleftarrow\joinrel\rhook y))$

4 Case study: citation as a cause-effect phenomenon

We consider here the following possible interpretation of the causality relation: $a \hookrightarrow b$ iff a is cited in b, where a and b are research papers. Research is an important activity of human beings and has been focused more and more on papers. So this example is relevant and up to date.

One may think this is strange, first by wondering if "papers" can be considered as events. What is a paper? We don't pretend here to study in details this question. But our analyzis may shed a side understanding of what a research paper is. First of all a research paper is not a material object, although it may manifest as such. We can roughly say that it is the expression of a research work. There is quite a variation surrounding this "object": a talk given at a conference, a powerpoint file, a preliminary version of the paper circulated among colleagues and friends, the galley-proofs of the paper, the on-line first version, already with an DOI identity number, and finally the "official" paper.

Considering as events only research papers, the relation of causality will be here only among papers. We do not deny that a paper may have been caused by the fly of a stork in a beautiful orange sky, or less poetically by a running board, to recall Poincaré's famous mystical experience [14]. But we consider only one aspect of reality. If a paper a is cited in another paper b, we may imagine that the work of (the authors of) a had some effect on the work of (the authors of) b, that in some sense it is a cause of b.

What are the properties of this causal relation? Let us present the following list:

- it obeys the principle of causality
- it does not obey the reverse principle of causality
- it is non-reflexive
- it is not antisymmetric
- it is not transitive
- it does not obey connectivity

Here are some comments. Principle of causality: a paper citing no other papers would be nowadays really weird. No reverse causality: many papers are not cited, we hope this will not be the case of the present one, that it will have many effects. Non-reflexivity: no comments. No antisymmetry: two papers are cited by each other, this happens and is explained by the process of production / edition of papers. No transitivity: that's reality! No connectivity: connectivity may appear when we restrict the field of research papers, for example considering papers on possible worlds.

Let us make now two remarks about conceptualization. We have here a good example where the notion of chain makes perfectly sense without transitivity. If several papers are cited in one given paper p, this does not mean that these papers have the same effect, at best we can say they have a common effect, p can be see rather as the "product" of these papers.

Acknowledgments and Personal Recollections

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I had been in contact with the ideas of Pat Suppes since a couple of years through his South American connection (Newton da Costa, Rolando Chuaqui, Francisco Doria) before I personally met Pat in Stanford in early 2000 where I was to stay about two years. After that I met him again in several places in the world: in Brazil, France, Switzerland. I organized jointly with Décio Krause a conference for his 80th birthday in Florianópolis in 2002. On the way to this magic island, he stopped in Rio de Janeiro were I was living at the time and we had a nice dinner at the Copacabana Palace with Acacio de Barros and other friends. In Paris we had lunch together with Anne Fagot-Largeault, who had worked with him at Stanford in the 1960s. I remember that we savored sardines directly in their cans in a restaurant on Boulevard Saint-Germain, much in the spirit of the French nouvelle cuisine. In 2004 Pat came to Bern, at the time I was working in the nearby town of Neuchâtel, to receive the Lauener prize, being the first recipient of this prize. We had a good time at this ceremony nicely organized by the manager of the Lauener foundation, Michael Frauchiger, with music and a high quality dinner.

Suppes was in fact a true *bon vivant*, enjoying good food and wine, cars, women, music, sport ... And he was doing research just because this is what he liked the most. When I was at Stanford, he was coming to Ventura Hall nearly every day, conducting seminars, directing the EPGY (Education Program for Gifted Youth) he had created and developing his Brain Lab. He was animating a group of people coming from all over the world.

I was working at the time on many different topics related to logic (universal logic, paraconsistent logic, modal logic, philosophy and history of logic). I presented at the seminar he was organizing with Dagfinn Føllesdal, a criticism of the rejection of propositions by Quine based on Suppes's congruency approach, see [2]. At some point he told me we should write a paper together. His proposal was to discuss and develop with me a general theoretico-philosophical perspective related to the experiments he was conducting on the brain with Marcos Perreau-Guimarães, a Brazilian-French guy he has just engaged. We had daily discussions on the basis on which I started to write my joint paper with Pat, "Semantic computation of truth based on associations already learned" [17]. We tried to develop in this paper a general framework for understanding the relations between language, reasoning and the brain. What is central is the description of a process of association using the axiomatic method. This illustrates a good lesson I learned from Pat Suppes: not to get stuck in some tricky details, but to focus one some fundamental general ideas, motivated by some deep philosophical reflections, developed in an abstract way, nonetheless always connected with reality.

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