Is Modern Logic Non-Aristotelian ?

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Abstract

In this paper we examine up to which point Modern logic can be qualified as non-Aristotelian. After clarifying the difference between logic as reasoning and logic as a theory of reasoning, we compare syllogistic with propositional and first-order logic. We touch the question of formal validity, variable and mathematization and we point out that Gentzen's cut-elimination theorem can be seen as the rejection of the central mechanism of syllogistic – the cut-rule having been first conceived as a *modus Barbara* by Hertz. We then examine the non-Aristotelian aspect of some non-classical logics, in particular paraconsistent logic. We argue that a paraconsistent negation can be seen as neo-Aristotelian since it corresponds to the notion of subcontrary in Boethius' square of opposition. We end by examining if the comparison promoted by Vasiliev between non-Aristotelian logic and non-Euclidian geometry makes sense.

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1. The two-stage history of logic

We can make a rough opposition between *Modern logic* and *Aristotelian logic*, saying that Aristotelian logic is the logic that has dominated from Aristotle up to the second part of XIXth when a lot of different changes appeared and we entered the area of Modern logic. From this point of view the history of logic is a two-stage story.

This is a common general vision of the situation but even if we agree with this perspective there are many things to examine and discuss. This is indeed a really complex matter directly connected to the understanding of the birth and development of modern logic. It is interesting to see what is radically different between the two stages and when and how the situation dramatically started to change.

We will focus here on the question to know in which sense Modern logic can be characterized as “non-Aristotelian” and what such an expression can mean. Let us note that even if many agree about this two-stage story, there is no common agreement about the terminology.

“Traditional logic” would be more neutral than “Aristotelian logic”, but since we want to focus on the question of the non-Aristotelian aspect of Modern Logic, we are using the expression “Aristotelian logic” for the first stage rather than “Traditional logic”. And we do that without ignoring that before Modern Logic there are trends of logic which can hardly be reduced to Aristotelian logic, not only in Oriental logic but also some trends of Occidental logic, like Stoic logic. Nevertheless, even taking in account all that, it is not erroneous to consider that Aristotelian logic is the main trend of logic before the mid XIXth century.

Our objective is not here to try to explain and understand what is on the one hand Aristotelian logic and on the other hand Modern logic – this would require thousands of pages - but to have some glimpses which may help us to have a better understanding of these two stages of logic by relating them.

2. Non-Aristotelian logic and Non-Aristotelian Logic

Comparing two things one can see differences and similarities. Starting from scratch one can say that Aristotelian logic and Modern logic are both *logic*. But what is logic? First let us note that Aristotle didn’t use the word “logic” as a name of a field, and also his main work on the topic has not been called *Logic* but *Organon*. It is not clear when exactly the word "logic" started to be used as a name of a field but the same problem exists also for "mathematics", "philosophy", "physics", … However for "logic" there is an additional problem since the word can be used in two different ways: the theory of reasoning and reasoning itself. This problem happens also for history, and we have proposed in a recent paper (Beziau 2010a) to follow the same scriptural distinction which can be described by the following tables:

History	**historical events**
history	**science of these events**

Logic	**reasoning**
logic	**science of reasoning**

When someone claims that Aristotle is the creator of logic, he certainly does not want to say that Aristotle was the creator of Logic, the first human being to reason. Aristotle himself by characterizing human beings as *logical animals* had the idea that logicality was an essential feature of human beings and had always been. When saying that Aristotle was the first logician and talking about "Aristotle's logic", one has clearly in mind a theory or conception of reasoning. The meaning of "logic" in the expression "Aristotelian logic" is more ambiguous and the ambiguity strongly grows when talking about "non-Aristotelian logic".

Korzybski (1933) and his followers, like the science-fiction writer A.E. van Vogt (1945), when promoting non-Aristotelian logic were rather referring to a new way of thinking or reasoning than to a new theory or a new system. On the other hand someone may develop a new theory of reasoning, a new system of logic, different from syllogistic, without wanting to change the way we are reasoning, but wanting to give a more accurate description of the reality of reasoning. This does not mean that this reality will change, but that we will have a different view of it, in the same way than conceiving physical reality from the point of view of Euclidean or non-Euclidean geometry will not change this reality (although it may change our interaction with it).

But logic is different from physics, it can make sense in logic to claim that (RC) *the reality of reasoning may change* and that (TRC) *a theory of reasoning may change our way of reasoning*. If one defends (TRC), one has to admit (RC), but one can also defend the idea that reasoning may change not necessarily by theorization. Someone may claim that reasoning "evolves" using some Darwinian theories. But we can also claim that reasoning may change without theorization of it and without biological change. And this is maybe what happened in Greece before Aristotle.

Before the reasoning theory of Aristotle was developed a new form of reasoning based on the use of the reduction to the absurd appeared. Some people consider this as the starting point of mathematics, because it was used to prove that the square root of two is irrational, which can be considered as the first important proof in the history of mathematics (see e.g. Dieudonné 1987).

To qualify this new way of reasoning as *Aristotelian* would be rather absurd because it appeared before Aristotle and Aristotle didn't give a clear account of it. We could simply qualify it as *logical*. One can argue that this change is a radical change in human mind. And in fact this makes sense if we think of the whole rationalist movement leading to the use of the reduction to the absurd (see Szabo 1969).

Aristotle's theory is certainly part of this rationalist movement, but his reasoning theory does not conceptualize the reduction to the absurd although he was the first to theorize the principle of contradiction. We now know that there is a strong relation between the reduction to the absurd and the principle of contradiction, but we also know the important differences between the reduction to the absurdum (which has two formulations) and some formulations of the principle of contradiction.

Let us have a look at the following table:

(RA)	IF $\neg P \vdash Q$ AND $\neg P \vdash \neg Q$ THEN $\vdash P$
(RA-)	IF $P \vdash Q$ AND $P \vdash \neg Q$ THEN $\vdash \neg P$
(CC)	$\vdash \neg(P \wedge \neg P)$
(VC)	$V(P)=1$ IFF $V(\neg P)=0$

We know today that (RA) implies – modulo some very elementary conditions - (RA-) and (CC) but is implied by none of them (for details see Beziau 1994). We also know that (VC) is equivalent to (RA), but this requires a proof that is not completely straightforward.

It would be rather ambiguous to call non-Aristotelian a Logic rejecting the reduction to the absurd such as the Logic of Brower - rejecting in fact (RA), but not (RA-). It is better to call it non-classical.

The expression “classical logic” is ambiguous (see Gourinat 2009). Depending on the way the word “logic” is used, it can be either interpreted as a classical theory of reasoning or as classical reasoning. And in both cases what does “classical” mean? It has at least three different meanings: (C1) related to ancient Greeks and Romans (C2) Related to a developed stage of a culture or civilization (C3) Standard and/or well-known. According to these three meanings it makes sense to say that the reasoning of Greek mathematicians is classical Logic. But the expression “classical logic” belongs to Modern logic and it is an ambiguous mix in which the word logic can be interpreted in two ways, this is not incoherent but rather confuse.

3. Syllogistic, Propositional Logic and First-Order Logic

Though Modern logic is quite different from Aristotelian logic, it was not mainly developed by a rejection of it. The work of two main figures of the development of Modern logic, Boole and Frege, can be considered as a continuation of the work of Aristotle. The work of Boole can be seen as a mathematization of syllogistic and Frege at the end of the *Begriffsschrift* presents the square of opposition, to show the harmony of his theory with the Aristotelian tradition. Boole and Frege’s systems appear therefore in a sense as improvements of the Aristotelian theory of reasoning.[1] Moreover Boole and Frege didn’t pretend to change the reality of reasoning, they were not proposing a non-Aristotelian Logic. On the other hand people like Henry Bradford Smith or Vasiliev who were using the expression “non-Aristotelian logic”, whether logic shall be understood here as reasoning or theory of reasoning, have had nearly no influence on the development of Modern logic.

We will not enter here in the details of the original works of Boole and Frege but we will discuss the core of Modern logic which they contributed in different ways to develop: propositional and first-order logic (hereafter POL and FOL). We will compare these systems with the core of Aristotelian logic,

[1] As Corcoran (2003, p.272) puts it: “The suggestion that Boole rejected Aristotle’s logical theory as incorrect is without merit or ground.”

syllogistic (hereafter SYL), which, with its figures and moods, can also be viewed as a system of logic.

Up to which point POL and FOL are fundamentally different from SYL and can be said to be non-Aristotelian? In Modern logic it is usual to consider that POL is the most elementary system of logic, sometimes considered as part of FOL. If one has an evolutionary view of history according to which complexity is increasing, one may think that Aristotelian logic is closer to POL than to FOL. But in fact most of the time first attempts are at the middle, not having the clarity of simplicity and not having the subtlety of complexity. SYL can be interpreted as something in between POL and FOL or a mix of them. But there are also some radical differences between SYL and both of them.

Let us first examine the relation between SYL and POL. In POL there are two kinds of objects: propositions and connectives. They can be interpreted in different ways, but let us stay as neutral as possible. Propositional logic is an abstract theory in the sense that abstraction is made of what there is inside the propositions, sometimes the expression “unanalyzed proposition” is used. It seems that Aristotelian logic didn’t reach this level of abstraction. In the *Categories*, which in the *Organon* is considered as a preliminary to SYL presented in the *Prior Analytics*, a proposition is presented as a combination of terms corresponding to predicates (“categories”). And the rules of SYL are based on what there is inside the three propositions constituting a syllogism: the major, minor and middle terms.

Today we can see these rules as interplay between quantifiers and negation. But this is a modern interpretation; quantifiers and negation do not appear in SYL as logical operators as conceived in Modern Logic. SYL can be interpreted as a logic of classes with classes and operations between classes as basic objects. To be more faithful, but speaking in a contemporary manner, it would be better to say that SYL is a logic of concepts extensionally conceived as classes.

Negation in Aristotelian logic doesn’t appear as connective as in Modern logic, something transforming a proposition into another proposition, and it would be difficult to argue that the concept of a binary connective can be found in Aristotelian logic. In SYL it does not appear. Even if we interpret a syllogism as a conditional, it is then an operator transforming true propositions in a true proposition. None of the 16 binary connectives and rules of POL explicitly appears in Aristotelian logic and for sure not in SYL.

We can therefore say that POL is not Aristotelian (as it is known Stoic logic is closer to POL, see Łukasiewicz 1927 and Gourinat 2000), but it would be difficult to claim that for this reason it is non-Aristotelian, because it is a conceptual and structural difference rather than an opposition.

None of the modern rules for quantifiers also appear in SYL. SYL can at best be interpreted from the perspective of first-order monadic logic (FOLM hereafter). Up to a certain point the figures of SYL can be seen as valid inferences of formulas of FOLM. But first of all this does not mean than we can generate the rules of FOLM from SYL. From the deductive point of view the figures of SYL together with the conversion rules is a small fragment of FOLM.

Moreover the structure of the formulas of FOLM is quite different from those of SYL. The original categorical propositions of Aristotelian logic have been wrongly interpreted in many different ways in the history of logic, in particular as propositions of type *S* is *P* where *S* is a subject and *P* is a predicate (see Heijenoort 1974). But categorical propositions are rather relations between two concepts.

A universal affirmative of the type "*A* belongs to all *B*", should better be interpreted as $B \subseteq A$ than as $\forall x(Bx \rightarrow Ax)$. In modern logic we can say that these two formulas are equivalent but from a conceptual point of view they are different. A formula like $\forall x(Bx \rightarrow Ax)$ has a level of abstraction that is very far from Aristotelian logic and which was conceived only in Modern Logic. Nevertheless Aristotelian logic reached also an important level of abstraction; this is what we will examine in the next section.

But before going into that let us look at SYL from the perspective of the following table:

A	***A*** **BELONGS TO ALL** ***B***	$B \subseteq A$
E	***A*** **BELONGS TO NO** ***B***	$A \otimes B$
I	***A*** **BELONGS TO SOME** ***B***	$A \, O \, B$
O	***A*** **DOES NOT BELONG TO SOME** ***B***	$B \not\subseteq A$

Let us recall that in standard mathematics $\subseteq$ is a relation not a function and an assertoric dimension is included into it. $\otimes$, O and $\not\subseteq$ in the above table should be understood in the same way, they are not functions like intersection. We can consider $B \not\subseteq A$ as the negation of $B \subseteq A$ and $A \otimes B$ as the negation of A O B but we have to keep in mind that negation of an asserted proposition is not a simple proposition, but an asserted proposition. From this point of view SYL systematically articulates the relation between two concepts.

To have a look at SYL in this sense gives a better understanding of its meaning than considering it as a fragment of FOLM or a fragment of a Boolean algebra of classes. Because in both of these cases it is not clear what is the meaning of this fragment and why not considering the whole system. But if we see SYL based on the table below, we can check that it is a complete systematization.[2]

4. Aristotelian logic, formal logic and mathematical logic

Despite some important different structural features between Aristotelian logic and Modern logic there is something which is common between them that can also be considered as structural, it is the fact that the validity of an argument is independent of the particular notions to which it applies. Validity is connected to some forms that can be applied (or in which can enter) many matters.

This central feature of reasoning is a particular case of Aristotle's general perspective of *hylemorphism* (cf. Largeault 1993). It is not clear in which sense the formal character of modern logic is linked to Aristotelian hylemorphism and what is the relation of hylemorphism with the notion of variable, a key notion for formalism. Aristotle uses variables but it would be difficult to argue that he reaches the idea of hylemorphism through the notion of variable. It is rather the other way round: Aristotle developed logical hylemorphism using variable. The formal character of reasoning, the validity of an argument not depending of its matter, is based in Aristotelian logic on some abstraction expressed by variables.

We are talking of "variables" but there is a big difference with informal use of variables as a notational procedure and the theory of quantification as it was

[2] Many papers are still been written about Syllogistic, here are two recent ones : (Alvarez-Fonticella 2016), (Murinová Nocák 2016), (Alvarez-Fonticella Correia 2016),.

developed in FOL.[3] The use by Aristotle of a certain notation, capital Greek letters to design arbitrary predicates, is a process of abstraction similar to the one in mathematics – maybe Aristotle was inspired by Greek mathematicians who were doing the same at this period, although the domain of variables are different in the two cases.[4]

The ambiguity of the relation between Aristotelian logic and mathematics appears when talking about *formal logic*. According to Scholz (1931) the expression "formal logic" has been introduced by Kant, and he was using it to talk about Aristotelian logic. However Modern logicians are sometimes using this expression to qualify Modern logic by opposition to Aristotelian logic without being aware of this fact. For many "formal" sounds like "mathematical", i.e. connected to the use of some *form*alism using *form*ulas, something which goes much further that Aristotle's use of capital letters as variables.[5]

In Modern logic there is also the formalist trend which has pushed to the extreme the general idea that the validity of an argument does not depend on the signification of signs but just on rules governing them which is not something necessarily mathematical and which can be seen as the continuation of Aristotelian logic.

But Modern logic does not reduce to the formalist trend and there have been people criticizing the very idea of formal validity. Wittgenstein is one of them but he didn't develop any system of logic. Such kinds of systems have been developed by relevantists (cf. Anderson and Belnap 1975). They use the word "relevant" to express that there is a connection between the premises and the conclusion of an argument. Technically speaking this has been developed through the condition that there must be some "contents" shared by the hypotheses and the conclusion. In propositional logic it is required that there is a least an atomic proposition common to the hypothesis and the conclusion. On this basis, the rule according to which from a proposition and its negation it is possible to deduce any proposition has been rejected. We can say that relevant logic is non-Aristotelian not because it rejects this rule – which is

[3] About the notion of variables in Aristotle's logic, see (Bocheński 1927, Łukasiewicz 1951, Corcoran 1974b, Westerstahl 1989, Smith 1974).

[4] About the use of variable in Greek mathematics and the difference with modern formalization, see (Vandoulakis 1998).

[5] We have discussed in details in another paper five variations of the meaning of "formal", see (Beziau 2008).

not explicit in Aristotelian logic – but because it rejects the idea that validity depends only on the form, and it emphasizes that we have to take in account the meaning of which about we are reasoning.

One of a striking features of Modern logic is its strong relation with mathematics. Due to this feature it would not be wrong to call Modern logic, "Mathematical logic", but there is an ambiguity since this expression can be interpreted as (ML1) the study of mathematical reasoning, (ML2) a mathematical study of reasoning, (ML3) a mathematical study of mathematical reasoning (about this last one, see Hintikka 2012). Modern logic oscillates between the three.

Aristotelian logic is a general theory of reasoning supposed to encompass all kinds of reasoning, including mathematical reasoning, but as it is known Aristotle had no special interest for mathematical reasoning and did not pay attention to it. That may explain why Aristotelian logic had absolutely no effect on the development of mathematics. The situation dramatically changed when people like Peano, Frege, Hilbert, Russell started to closely examine mathematical reasoning leading to *foundations of mathematics*, *principles of mathematics*, *metamathematics* which are important parts of Modern Logic and which can in some sense all be considered as not Aristotelian (rather as non-Aristotelian).

On the other hand Modern logic does not reduce to this trend, another trend is the one originated by Boole: the use of mathematics to understand reasoning. Boole's approach to logic is Aristotelian in the sense that he is considering any kinds of reasonings but his methodology is different since he started to systematically use mathematics for doing that and was the first to do it as stressed by Corcoran (2003, p.261): "In *Laws of Thought Boole* presented the world's first mathematical treatment of logic."

5. Farewell to Barbara

Modern logic is a fascinating mix - using mathematics to understand mathematical reasoning - which led to some astonishing results like the cut-elimination theorem, one of the central and most important results of Modern logic proved by Gentzen.

Gentzen's cut-rule is directly inspired by a rule of Hertz that Hertz was considering as a formulation of the syllogism of Barbara, the most famous and typical rule of syllogistic (that's why Hertz sometimes simply calls his rule "Syllogismus" – see Hertz, 1931) . Hertz was using this rule as a basic rule for a general abstract and structuralist theory of deduction called *Satsysteme*, a step towards universal logic - see (Legris 2012), a presentation of the first English translation of Hertz's work in (Beziau 2012).

Gentzen then constructed a logical system now called *sequent calculus* with on the one hand an adaptation of the abstract rules of Hertz including the *Syllogismus* rule he renamed with the sharper name "cut" ("Schnitt" in German) and on the other hand rules for connectives and quantifiers. All the rules of SYL have a common feature with the Barbara rule: there is a term which is disappearing, the middle term. And in fact this is the main plot of SYL, by cutting the middle term a fatal conclusion is reached. To see things less tragically but still dramatically we can say that reasoning in Aristotelian logic can be viewed as establishing a connection between two notions through a common one that is a bridge between them, the bridge explodes once it has been crossed.

Gentzen constructed a system where this elimination phenomenon is concentrated in only one rule, the cut-rule, and showed that we can get the same results with the system with the cut-rule and the system without it, so that these two systems are equivalent (he did that both for classical logic and intutionistic logic). To show that he performed a sophisticated double recurrence reasoning, probably the first in the history of mathematics.[6]

One of the most important consequences of Gentzen's theorem is the relative consistency of arithmetic that he proved just after the famous negative result of Gödel. This is a very important result from the point of view of metamathematics but the cut-elimination theorem, largely ignored by philosophers, is also a very significant result for philosophy. It means that logical truth (conceived and/or described in the perspective of classical logic, intuitionistic logic and a great variety of logical systems) is analytic in the sense that all we need to prove the validity of a theorem is included in the formulas expressing the theorems.

The cut-elimination theorem is a very challenging result completely opposed to the picture of reasoning given by SYL. We can claim that Gentzen's

[6] It would be possible to argue that performing such reasoning is a new advance in Logic, similar to the one corresponding to the apparition of the reduction to the absurd. More generally it would make sense to say that Modern logic presents many aspects of a new Logic, with proofs such as diagonalization and so on.

system LK without cut is really anti-Aristotelian (and the same for other systems without cut).

6. Are non-classical logics non-Aristotelian?

Up to now we have mainly spoken about some structural aspects of logic, another perspective is a "principle perspective". Does Modern logic reject some basic principles or laws of logic admitted by Aristotle?

By contrast to Aristotelian logic, which is rather monolithic, in Modern logic we have many different systems of logic, ranging from extensions of "classical logic" to deviations which are called "non-classical logics" (see Beziau 2015a).

Among the extensions, the most famous ones are modal logics. It is known that Aristotelian logic is strongly connected to modalities (see Łukasiewicz 1951, Patterson 1995, Rini 2011) and therefore the modal perspective in Modern Logic is in some sense Aristotelian. But there is the question to know if the approach is the same, in particular if the systems of modal logic in Modern Logic conform to Aristotle's views of modality. Today a modality like necessity appears as a logical operator within a system of logic but it can also be considered as a metalogical level and there is the question of the interplay between these two levels (see Beziau 2013), to which is related the question to know if a rule like necessitation makes sense or not (see Łukasiewicz 1953). Someone may wonder up to which point modern modal logic is Aristotelian or not, but we will not develop this point here; we will focus on the rejection of some basic principles of logic.[7]

What are the fundamental principles or laws of logic? Before Modern logic, the following five principles were considered:

(ID)	PRINCIPLE OF IDENTITY
(RS)	PRINCIPLE OF SUFFICIENT REASON
(BI)	PRINCIPLE OF BIVALENCE
(CO)	PRINCIPLE OF CONTRADICTION
(EM)	PRINCIPLE OF EXCLUDED MIDDLE

[7] We are aware that there are thousands of non-classical logics but in many cases it does not really make sense to examine if they are non-Aristotelian or not, what is obvious is that they are different from Aristotelian logic.

Only the three last ones clearly appear in Aristotelian logic,[8] so we will focus our discussion here on them, examining in which sense they can be considered as supported by Aristotelian logic and rejected by Modern logic.

Some people have the idea that these three principles are fundamental principles of Aristotelian logic. This trinity is even considered as the basis of occidental culture – Aristotelian logic being a symbol of it - by opposition to oriental culture. But this mythology is surrounded by a lot of misunderstandings. It can be in fact argued that Aristotle didn't absolutely defend any of these principles. There are different ways to formulate these principles and the ambiguity of the mythology is connected to the fuzziness of these formulations.

The principle of bivalence can be formulated as: (BI) *a proposition is either true or false*. This formulation is ambiguous. (BI) can be decomposed in two principles and this is useful to avoid ambiguity. The situation can be clarified by the following table where (BI) is considered as the conjunction of (B1) and (B2):

<table>
<tr><td rowspan="2">(BI) A PROPOSITION IS EITHER TRUE OR FALSE</td><td>(B1) A PROPOSITION CANNOT BE NEITHER TRUE NOR FALSE</td></tr>
<tr><td>(B2) A PROPOSITION CANNOT BE BOTH TRUE AND FALSE</td></tr>
<tr><td colspan="2">PRINCIPLE OF BIVALENCE BI = B1 + B2</td></tr>
</table>

Some people are identifying (B1) with (EM) and (B2) with (CO) and from this point of view (BI) appears as the conjunction of (EM) and (CO). But in Modern logic we can have a formulation of these principles according to which (BI) may hold and (EC) and (CO) are not valid.[9]

In Aristotelian logic it seems that (B2) is admitted but not (B1). So independently of interpreting (B1) as (EM) or not, we can say that Aristotelian reject bivalence. The fact that a proposition can be neither true nor false has been systematized in Modern Logic with three-valued logic by Łukasiewicz

[8] As it is known (RS) - *Nihil est sine ratione* - was introduced only in the middle age. About (ID), Bocheński (1951, p.43) wrote: "We find no principle of identity in the preserved writings of Aristotle".
[9] We have developed a detailed discussion about this in (Beziau 2003) that we will not repeat here.

(1920), one of the main promoters of many-valued logic, introducing a third value called "undetermined" or "possible" in connection with a problem discussed by Aristotle, the so-called *future contingents.* Many-valued logic can therefore be seen as in the spirit of Aristotelian logic. Many-valued logics have not been called "non-Aristotelian", but "non-Chrysippian" (see Moisil 1972), Chrysippus being considered as the defender of the principle of bivalence, not Aristotle.

Using the modern technology of logical matrices (expressed by truth-tables) it is quite easy to construct a logic with three values – a logic therefore derogating (B1) - in which the proposition $p \vee \neg p$ is valid. And it also possible to construct a logic using a non-truth-functional bivalent semantics in which $p \vee \neg p$ is not valid. In a certain way it makes no sense to discuss if in Aristotelian logic the formula $p \vee \neg p$ is valid or not because, as we have said, there is no theory of connectives in Aristotelian logic, and also all this question of truth-table, truth-functionality is beyond the scope of Aristotelian logic. But we can make the distinction between (B1) and (EM) without explicitly formalizing (EM) as the validity of $p \vee \neg p$ and without considering negation as a connective. Aristotle is known to have promoted the idea of contrariety (see Beziau 2016a and Lachance 2016) of which the trichotomy true-false-undetermined can be seen as an example rather than the other way round. In this sense we can say that Aristotle was also rejecting (EM): some opposed pairs of predicates or propositions admit a third element beyond them. This is related to the theme of the square of opposition to which we will come back later.

The situation with (B2) and (CO) is similar (dual can we say) but trickier due to the heavy mythology surrounding the notion of contradiction. In Modern logic, there is a clear name for logics rejecting (B1) , they are called *many-valued logics* ("many" being understood as more than two – see Beziau 1997 for a detailed discussion of this point), but there is no clear name for logics rejecting (B2), logics in which a proposition can both be true and false. *Paraconsistent logics* are generally presented as logics rejecting (CO), not (B2), and in fact no logical systems has been constructed in which (B2) is not valid because in Modern logic the relation between propositions and truth-values is considered as a function (see Beziau 2010b). This means that by definition it is not possible to attribute more than one value to a proposition. Philosophers sometimes want to promote some very challenging ideas in logic but at the same time for developing them they are rather conformist, presenting systems which from the mathematical point of view are rather reformed systems, than revolutionary systems.

Some paraconsistent logics have been constructed in which the idea of being truth and false is represented by a third value different from truth and falsity which is called "true-false" (cf. Priest 1979 – sometimes this third value is presented as a set). This is a rather paradoxical name because in this case a proposition which is true-false is neither false nor true, like in other many-valued logics. Anyway, similarly to (EM) and (B1), (CO) and (B2) are independent from the modern viewpoint. So one may focus on rejecting (CO) and this is what have been doing the paraconsistentists.

But what is exactly the principle of contradiction (CO)? The central idea of paraconsistent logic is to reject $p, \neg p \vdash q$ called, among other names, *principle of explosion* (EX). But is (EX) - or an informal presentation of it - a formulation of (CO)? Aristotle didn't present (CO) in this way. His formulation is closer to two other modern formulations: (CC) and (VSC), that we present here in a table summarizing the variety of formulations of (CO):

(B2)	**A PROPOSITION CANNOT BOTH BE TRUE AND FALSE**
(EX)	$P, \neg P \vdash Q$
(CC)	$\vdash \neg(P \wedge \neg P)$
(VSC)	**IF** $V(P)=1$ **THEN** $V(\neg P)=0$
PRINCIPLE OF CONTRADICTION	

In Modern logic, from the viewpoint of the theory of valuation (see da Costa/Beziau 1994), (VSC) can be seen as equivalent to (EX), but both are different from (CC). (CC) and (EX) are independent from each other as it can be shown using three-valued matrices (for a recent study about this, see Beziau/Franceschetto 2015, Beziau 2016b).

Aristotle has put a strong emphasis on the principle of non-contradiction, more than anybody he has promoted this principle as a central principle of logic. Łukasiewicz (1910) in a pivotal work has criticized the arguments of Aristotle in favor of the principle of non-contradiction. However Łukasiewicz didn't develop systems of paraconsistent logic. Such systems were developed later on in particular by Newton da Costa who promoted the terminology "paraconsistent logic" – the name was coined by his friend Miró Quesda (see Costa, Beziau, Bueno, 1993). Da Costa in his seminal book (1980) has himself examined Łukasiewicz's criticisms of Aristotle (see also Seddon, 1996 and Raspa 1999).

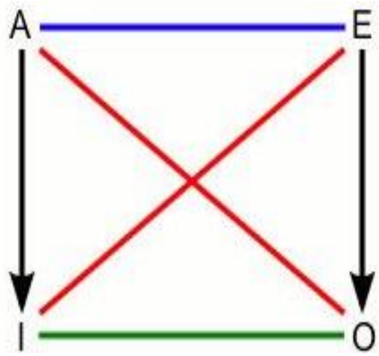
Even if one agrees that for Aristotle the principle of non-contradiction was very important, one may argue that paraconsistent logic is not radically anti-Aristotelian for two reasons. The first is that the principle of non-contradiction does not appear as a fundamental principle of syllogistic, and that in some sense syllogistic may applied to contradictory propositions (see Gomes and D'Ottaviano, 2010). The second reason has to do with the notion of subcontariety that appears in the square of opposition, according to which two opposed propositions can both be true. But we have here to make a clear distinction between Aristotle and neo-Aristotelian logic, this is what we will discuss in the next section.

7. The square of opposition and neo-Aristotelian logic

It is interesting to study the fate of the square of opposition to see the relations between Aristotelian logic and Modern logic. First of all let us point out the square of opposition is a good "symbol" of the ambiguity of the expression Aristotelian logic which ranges from the original theory of Aristotle through many adaptations and transformations which can be seen as improvements or deformations. At some point it is important to make a sharp distinction expressed by two different terminologies "Aristotelian logic" and "neo-Aristotelian logic", to which can be added a third one, "Aristotle's logic", if one wants to concentrate on the original doctrine of the Stagirite.

The "classical" formulation of the square is due to Boethius (see Correia 2012). The standard formulation of the square in Modern logic is an interpretation of Boethius's square in classical monadic first-order logic. Due to the question of existential import, one may reject the square, saying that modern logic, in particular first-order classical logic is non-Aristotelian, considering the square as typically Aristotelian. There are many ambiguities in this description: (SQ1) Boethius's square is typically neo-Aristotelian; (SQ2) the question of existential import was discussed before modern logic; (SQ3) Frege considered that his theory of quantification fitted in the square; (SQ4) It is easy to find modern abstract versions of the square which makes sense. We will not discuss in the present paper all these questions, the reader may have a look at recent literature on the subject (Beziau & Payette 2008, 2012, Beziau & Jacquette 2012, Beziau & Read 2014, Beziau & Gerogiorgakis, 2016, Beziau & Basti 2016).

Let us first recall that Aristotle didn't explicitly draw any square (although he suggested such a figure, see Horn 2015), but developed some ideas which lead to a theory which was represented many centuries later by a picture, first by Apuleius and then by Boethius (see Correia 2016 about the relation between the two). The square is an insightful way to represent the relations between the four types of categorical propositions. Here is a colored picture of it:



We have represented the relation of *contrariety* in blue, the relation of *subcontariety* in green and the relation of *contradiction* in red. In black is the notion of subalternation which is, as the arrow indicated, a kind of implication. The three notions of oppositions are defined as follows:

(CS) Contradiction ***p*** **and** ***q*** **cannot both be true together and cannot be both false together**	**(C) Contrariety** ***p*** **and** ***q*** **can be false (but not true) together**
	(S) Subontrariety ***p*** **and** ***q*** **can be true (but not false) together**
THE THREE OPPOSITIONS OF THE SQUARE	

In the picture of the square, the notion of subcontrariety appears as dual of the notion of contrariety and the theory exposed in this way permits to defend the idea that subcontariety is as much an opposition as contrariety, though originally Aristotle didn't consider it as an opposition.

The three notions of oppositions as defined in the above table do not depend on negation within propositions contrarily as in Boethius's square or other representations of the square through particular cases of propositions.

If we consider the definition of contradiction as given in the above table, we can see that only a negation obeying the principle (VC) $v(p)=1$ iff $v(\neg p)=0$ defines pairs of contradictory propositions. So it makes sense from this perspective to say that *a negation obeys the principle of contradiction* only if it obeys (VC). If we define a paraconsistent negation as a negation no obeying (EX) $p, \neg p \vdash q$ then it does not obey (VC), therefore we can say that a paraconsistent negation does not obey the principle of contradiction.

Contrarily to what Slater (1995) claimed, it can make sense to defend the idea that a paraconsistent negation is a negation and this thesis can be supported by the square of opposition, since in the square we have three notions of oppositions and we can argue that to these three notions of oppositions correspond three notions of negation (Beziau 2003b). In this sense the idea of paraconsistent negation is Aristotelian or to be more exact neo-Aristotelian. On the other hand from this perspective the idea of *true contradiction* does not make sense: if we have as proposition such that $v(p)=v(\neg p)=1$, the pair p and $\neg p$ is not a contradiction (see Beziau 2015b and Becker Arenhart, J.R.: 2016).

One may say that paraconsistent logic are logics derogating the principle of contradiction meaning that in these logics it is possible to define a negation not obeying the principle of contradiction considered as (VC). Such a negation is not necessarily anti-Aristotelian, because it can be considered as corresponding to the notion of subcontariety (S), but there are also some paraconsistent negations not corresponding to (S) in particular those who are at the same time derogating (S) and (C), which have be called paranormal and are exemplified in a simple logic system, called De Morgan logic (not due to De Morgan, the expression was coined by Moisil).[10] A paranormal negation is not neo-Aristotelian in the sense that it does not fit in the square, to claim that it is non-Aristotelian is another story.

[10] The terminology « paranormal negation » was introduced by Beziau; De Morgan logic emerged from De Morgan algebra (for details about that see Beziau 2012c, 2012d).

8. Non-Aristotelian logic and Non-Euclidian geometry

It is not an exaggeration to say that Vasiliev is the only person who has used the expression "non-Aristotelian logic" in a reasonable way. Besides him, we have on the one hand some people like Henry Bradford Smith (1918) who have used this expression to talk about variations on Aristotelian logic, and on the other hand people like Korzybski (1933) and his followers who have used it in a sense premonitory to new age and postmodernism.

Vasiliev's use of this word is strongly linked to an analogy with Non-Euclidian geometry and is directly inspired by it. Valisiev is not using only the expression "non-Aristotelian logic" but also the expression "Imaginary logic" considered as equivalent to it in the same way that in geometry the two expressions "non-Euclidian geometry" and "Imaginary geometry" are used.

Nicolai Alexandrovich Vasiliev (1880-1940) was from Kazan like Lobachevski and his family was linked to him. V.Bazhanov has extensively written about the life and work of Vasiliev (see details in the bibliography). Vasiliev is the author of three main papers, all in Russian, one of them called "Imaginary (non-Aristotelian) logic" (Vasiliev 1912) which has been translated into English as well as the third one (Vasiliev 1913) entitled "Logic and metalogic". The first one (Vasiliev 1910) has not been translated into in English but partial translation in Portuguese of it is available in a booklet entitled *N.A.Vasiliev e a lógica paraconsistente* (Arruda 1990) which is a condensed translation of these three main papers of Vasiliev produced by A.I.Arruda, student and colleague of Newton da Costa. They were the first to stress the relation between Vasiliev's ideas and paraconsistent logic. The objective of our present paper is not to present a detailed account of Vasiliev's ideas, we will focus here on the analogy between non-Euclidian geometry (NEG hereafter) and "non-Aristotelian logic" (NAL hereafter) developed by Vasiliev in his three main works.

The initial idea of Vasilev seems for us today very simple: his main point of departure is that the situation is similar between NEG and NAL in the sense that as we can modify an axiom of geometry, we can modify an axiom of logic. Vasiliev is talking of a special axiom, but before considering that, we can examine the similarity of the methodology independently of this special axiom. This idea of Vasiliev looks obvious to us today because we are acquainted to the generalized axiomatic method, developed in particular by Hilbert, applying to logic itself considered as a collection of systems among others. But at the very beginning of the XXth when Vasiliev was developing his ideas this was far to be obvious. Hilbert published his seminal work about the axiomatization of geometry in 1899 and as it is known someone like Frege didn't contemplate this axiomatic vision (see e.g. Resnik 1973).

What Vasiliev defends is the very idea of applying the axiomatic method to logical systems: "By means of this method one could, as it seems, first of all more reliably determine the axioms and postulates that constitute the foundation of logic; secondly, one could give them precise formulations, since by enumerating all the axioms we could avoid the constant recurring conflation of different axioms; thirdly, one could demonstrate that all the axioms discovered are independent and are not derived from each other, since independence is a basic property of the concept of an axiom or a basic principle; fourthly, one could determine which logical operations and propositions depend upon which axiom (for example, when, upon removal of the axiom, these operations themselves have to be abolished); finally, one could formulate a complete classification or system of axioms and postulates for logic. In short, for logic the same kind of investigation should be carried out that has already been carried out for geometry, viz. an axiomatic one" (Vasiliev 1912 p.162)[11]. It is worth recalling that this methodology was applied in particular by the close collaborator of Hilbert, Paul Bernays, in his habilitation thesis defended in 1918; Bernays used three-valued matrices to study the independence of axioms of classical propositional logic.[12]

Vasiliev's ideas were in the spirit of the Hilbert's school, Vasliev was aware of the work of Hilbert but didn't know the details. Vasiliev had few knowledge of the specific advances of Modern logic, he was still merged in categorical propositions and syllogism. When talking about the axioms of logic he refers to something connected with the table we have presented in section 6, some Aristotelian and neo-Aristotelian axioms. He considers 4 axioms: (ID), (CO), (EM) and (RS) and for him these are laws of thought (Vasiliev 1912, pp.128-129). He doesn't directly consider the principle of bivalence (BI), but considers the second part of it (B2) calling it *the law of the absolute difference between*

[11] We indicate here the date of the original text, which is important for the discussion, but the page number is the one of the English translation indicated in the bibliography.

[12] This work has never yet been translated in English but recently was published an English translation of the published paper which is an abridged version of it (Bernays 1926) presented by Carnielli (2012).

truth and falsity - a fundamental law that cannot be rejected which is qualified as metalogical – to which we will come back later.

Vasiliev makes a “parallel” between (CO) and the axiom of parallel in geometry, arguing that these axioms in both cases can be withdrawn: “Non-Euclidean geometry is a geometry without the 5th postulate, [that is] without the so-called axiom of parallels. Non-Aristotelian logic is a logic without the law of contradiction. It is worth mentioning here that it was precisely non-Euclidean geometry that has served us as a model for the construction of non-Aristotelian logic.” (Vasiliev, 1912, p.128).

According to Vasiliev this axiomatic construction is non-Aristotelian because it is opposed to the Aristotelian logic describing our world and it is imaginary because it describes an imaginary world: “Our logic is the logic of reality, in the sense that it is a tool for knowledge of this reality, and thus is closely connected with it. The new logic does not have such a connection with our reality; it is a purely ideal construction. Only in a world different from ours, in an imaginary world (the basic properties of which we can, nevertheless, exactly define) imaginary logic could be a tool for knowledge” (Vasiliev 1912, p.127)

When talking about worlds, Vasiliev is talking about the Earth and other planets and he thinks that (CO) is empirical: it is an axiom that rightly describes the situation on Earth but he argues that there may be worlds in which it does not apply and that we can imagine this kind of worlds by withdrawing (CO) in the same way that by withdrawing the axiom of parallel in NEG we have access to imaginary worlds. This view seems to us nowadays quite exotic, but we have to remember that at this time people had a completely different vision of the universe and that the theory of relativity based on NEG was just being born.

Today the idea is not really to consider that the physical laws or logical laws are different in different planets. Even in possible worlds semantics all the worlds have the same logic, not necessarily the classical one, but a collection of worlds with different logics is not considered. By opposition to Vasiliev some people in contemporary logic are considering that the logic of physical reality and/or the logic of our thought are not necessarily classical. And it is not clear that alternative logics can be properly characterized as non-Aristotelian. As we have said, Brouwer’s intuitionistic logic can be rather considered as rejecting the reduction to the absurd, a pre-Aristotelian reasoning and something like quantum logic rejecting the law of distributivity rejects a principle which was not conceived or formulate by Aristotle.

For Vasiliev there are three dimensions of logic:

EARTHLY LOGIC	ARISTOTELIAN, EMPIRICAL, THE LOGIC OF EARTH
IMAGINARY LOGIC	ANY LOGICAL SYSTEMS, APPLYING TO IMAGINARY WORLD OR OTHER PLANETS
METALOGIC	THE LOGIC OF THE FORM OF OUR THOUGHT, ABSTRACT AND NON-EMPIRICAL
THREE ASPECTS OF LOGIC - VASILIEV	

For Vasiliev though Metalogic is different from Earthly logic because it is not empirical, it obeys the same rules. The idea that metalogic is “classical” by difference to non-Aristotelian imaginary logic is not without some problems (how can we imagine a non-classical world/logic from the perspective of the classical world we are merge in?) but it has been widely adopted in Modern logic in the sense that it is a standard position to consider that the metalogic of non-classical logics is classical.

However the parallel here between logic and geometry is not clear. Vasiliev makes the following comparison which is not really convincing: “There should be logical truths which follow from the very definition of the logical, which are of absolute validity for any logic, for any logical thinking. If we find anywhere a consciousness without these truths, we shall simply say 'It is not logical', but not 'It has another logic'. Just as if we find a geometry without three-angled figures we shall say 'In this geometry there are no triangles' but for this reason the truth 'All triangles have three angles' does not cease to be true.” (Vasiliev 1913, p.331).

In Modern logic B2 is distinct from other formulations of the principle of contradiction – see the table in section 6. Vasiliev insists on the difference between B2 and what he calls the principle of contradiction and in fact we can agree with him not to call B2 the principle of contradiction. As we have seen he called it “the law of absolute difference between truth and falsity”: “In order to avoid any misunderstanding, it is necessary to distinguish now between the rejected law of contradiction and another one which is (sometimes) confused with it and which cannot be rejected. We would like to call this law the law of absolute difference between truth and falsehood, which can be formulated as follows: 'One and the same proposition cannot be true and false simultaneously'. It is impossible to reject this law, since anyone who would reject it, and therefore confuse truth and falsehood, would stop to reason logically at all. Therefore, this law remains valid in imaginary logic as well. (Vasiliev 1912, p.136)

Even if nowadays, as we have stressed, paraconsistent logic does not derogate B2 it is not clear that B2 is an absolute principle of thought. One could argue (PL1) that this principle does not describe the way our thought actually works, (PL2) that maybe our thought usually works like that but we may change this and create a new Logic. Moreover it would not be difficult to construct a mathematical system derogating this law (but if we want this not to be just a formal game, we should philosophically defend it).

What Vasiliev calls the principle of contradiction, by opposition to B2, is not really clear, he insists on the fact that besides affirmation and negation, there is a third situation that he calls "indifferent". This makes us think of neither-true-or-false rather than both-true-and-false, in any case as something like three-valued logic and moreover Vasiliev defends a law of *excluded fourth*. It has been discussed whether Vasiliev was the forerunner of many-valud logic or paraconsistent logic. Some people have argued that Vasiliev had wrongly been considered as the forerunner of the former, that he should be rather considered as a forerunner of the latter. But in view of three-valued parconsistent logic - developed by Asenjo (1966), D'Ottaviano and da Costa (1970), Priest (1979) - this makes sense to consider him as a forerunner of both since the two are interlinked (but paraconsistent logic can be developed outside of three-valued logic).

As we have seen paraconsistent logic is not something which is necessarily dramatically non-Aristotelian, on the other hand many paraconsistentists consider that paraconsistent logic is the logic of our world and/or thought not a logic of an imaginary world. The only thing left is the starting point of Vasiliev, the application of the modern axiomatic method to logic, which is difficult to consider by itself as leading to something that can be characterize as non-

Aristotelian logic, although we can consider that it can lead to many different conceptions of logic and the world.

9. Bibliography

Ables, F. and Fuller, M.E. (eds): “Modern Logic 1850-1950, East and West”, Birkhäuser, Basel, 2016.

Alvarez-Fontecilla, E., 2016, “Canonical Syllogistic Moods in Traditional Aristotelian Logic”, *Logica Universalis*, **10** (2016).

Alvarez-Fontecilla, E. and Correia, M.: 2016, “Conversion and opposition: traditional and theoretical formulations” in Beziau & Gerogiorgakis 2016.

Anderson, A.R. and N.Nuel, 1975, *Entailment, the Logic of Relevance and Necessity*, vol. I, Princeton University Press, Princeton.

Arruda, A.I. 1990 (ed): *N.A.Vasiliev e a lógica paraconsistente*, CLE, Campinas.

Asenjo, F.: 1966, “A calculus of antinomies”, *Notre Dame Journal of Formal Logic* **7** (1966), pp.103-115.

Bazhanov, V.A.: 1990, “The fate of one forgotten Idea: N.A. Vasiliev and his imaginary logic”, *Studies in Soviet Thought*, **39** (1990) pp. 333-344.

Bazhanov, V.A.: 1993, “C.S.Peirce’s Influence on Logical Ideas of N.A. Vasiliev”, *Modern Logic*, **3** (1993), pp. 45-51.

Bazhanov, V.A.: 1994, “The Imaginary Geometry of N.I. Lobachevsky and the Imaginary Logic of N.A. Vasiliev”, *Modern Logic*, **4** (1994), pp. 148-156.

Bazhanov, V.A.: 1998, “Toward the Reconstruction of the Early History of Paraconsistent Logic: the Prerequisites of N. A. Vasiliev’s Imaginary Logic”, *Logique et Analyse*, **161-63**, pp. 17 – 20.

Bazhanov, V.: 2001, “The origins and becoming of non-classical logic In Russia (*XIX* - the turn of *XX* century)”, in *Zwischen traditioneller und moderner Logik. Nichtklassiche Ansatze*. Mentis-Verlag, Paderborn, pp.205-217.

Bazhanov, V.A.: 2008, “Non-classical stems from classical: N. A. Vasiliev’s approach to logic and his reassessment of the square of proposition”, *Logica Universalis*, **2** (2008), pp. 71-76.

Bazhanov, V.A. : 2011, “The dawn of paraconsistency: Russia’s logical thought in the turn of XXth Century”, *Manuscrito*, **34**, pp. 89-98.

Bazhanov, V.A. : 2016, “Russian origins of non-classical logics” in (F.Abeles and M.Fuller 2016), pp.197-203.

Bazhanov, V.A. : 2016b, “Nicolai Alexandrovich Vasiliev”, *Internet Encyclopedia of Philosophy.*

Becker Arenhart, J.R.: 2016, “Liberating paraconsistency from contradiction*”, Logica Universalis*, **10** (2016), pp.523-544.

Bernays, P.: 1926, “Axiomatische Untersuchungen des Aussagen Kalküls der *Principia Mathematica*”, *Mathematische Zeitschrift*, **25**, pp.305-320, translated as “Axiomatic investigations of the propositional calculus of the *Principia Mathematica*" in (Beziau 2012a), p. 43-57.

Beziau, J.-Y.: 1994, “Théorie législative de la négation pure”, Logique et Analyse, **147-148** (1994), pp.209-225.

Beziau, J.-Y.: 1997, “What is many-valued logic ?”, in *Proceedings of the 27th International Symposium on Multiple-Valued Logic,* IEEE Computer Society, Los Alamitos, pp.117-121.

Beziau, J.-Y.: 2003a, “Bivalence, exluded middle and non contradiction’, in *The Logica Yearbook 2003*, L.Behounek (ed), Academy of Sciences, Prague, pp. 73-84.

Beziau, J.-Y.: 2003b, “New light on the square of oppositions and its nameless corner”, *Logical Investigations*, **10** (2003), pp.218-232.

Beziau, J.-Y.: 2008, “What is "formal logic" ?”, in Myung-Hyun-Lee (ed), *Proceedings of the XXII World Congress of Philosophy*, vol.13, Korean Philosophical Association, Seoul, pp.9-22.

Beziau, J.-Y.: 2010a, “Logic is not logic”, *Abstracta***, 6** (2010), pp.73-102

Beziau, J.-Y. : 2010b, “Truth as a mathematical object”, *Principia*, **14**, pp.31-46.

Beziau, J.-Y. (ed): 2012a, *Universal Logic : an Anthology*, Birkhäuser, Basel.

Beziau, J.-Y.: 2012b, “History of truth-values", in D.M.Gabbay and J.Woods (eds) *Handbook of the History of Logic, Vol. 11 - Logic: a history of its central concepts,* Elsevier, Amsterdam, pp.233-305.

Beziau, J.-Y.: 2012b, “Paralogics and the theory of valuation”, in (Beziau 2012a), pp.361-372.

Beziau, J.-Y.: 2013, “The metalogical hexagon”, *Argumentos* , 10 (2013), pp.111-122.

Beziau, J.-Y. : 2015a, “The relativity and universality of logic”, *Synthese*, 192 (2015), pp.1939-1954.

Beziau, J.-Y. : 2015b, “Round squares are no contradictions”, in *New directions in paraconsistent logic*, Springer, New Dehli, 131-145.

Beziau, J.-Y.: 2016a, “Disentangling Contradiction from Contrariety via Incompatibility”, *Logica Universalis*, 10 (2016), pp.157-170.

Beziau, J.-Y.: 2016b, “Two Genuine 3-Valued Paraconsistent Logics”, in S.Akama (ed) *Towards Paraconsistent Engineering* Springer, Heidelberg, 2016, pp.35-47

Beziau, J.-Y. and Franceschetto, A.: 2015, “Strong three-valued paraconsistent logics”, in *New directions in paraconsistent logic*, Springer, New Dehli, 131-145.

Beziau, J.-Y. and D.Jacquette (eds): 2012, *Around and beyond the square of opposition*, Birkhäuser, Basel, 2012.

Beziau, J.-Y. and G.Payette (eds): 2008, *The square of opposition,* Special Issue of *Logica Universalis*, issue **1**, volume **2**.

Beziau, J.-Y. and G.Payette (eds): 2012, *The square of opposition - A general framework for cognition*, Peter Lang, Bern.

Beziau, J.-Y. and S.Read (eds) : 2014, *The square of opposition in historical perspective*, special issue of *History and Philosophy of Logic,* issue **3**, volume **35**.

Beziau, J.-Y. and S.Gerogiorgakis (eds): 2016, *New dimensions of the square of opposition*, Philosophia Verlag, Munich.

Beziau, J.-Y. and G.Basti (eds): 2016, *The square of opposition: A cornerstone of thought*, Birkhäuser, Basel.

Bocheński, J-M.: 1927, “On logical relativism”, translated an reprinted in *Axiomathes*, 2, 1993, pp. 193-209.

Bocheński, J-M.: 1951, *Ancient formal logic*, North Holland, Amsterdam.

Boole, G.: 1854, *An Investigation of the laws of thought,_on which are founded the mathematical theories of logic and probabilities,* Macmillan & Co., London, and Cambridge.

Carnielli, W.A.: 2012a, “Paul Bernays and the Eve of Non-standard Models in Logic” in (Beziau 2012), p. 33-42.

Corcoran, J.: 1973, “A Mathematical Model of Aristotle's Syllogistic”, *Archiv für Geschichte der Philosophie*, **55**, 191–219.

Corcoran, J.: 1974, *Ancient logic and its modern interpretations*, Proceedings of the Buffalo Symposium on Modernist Interpretations of Ancient Logic, Reidel., Dordrecht.

Corcoran, J.: 1974b, “Aristotle's Natural Deduction System”, in (Corcoran 1974), pp.85-132.

Corcoran, J.: 1974c, “Aristotelian Syllogisms: Valid arguments or true generalized conditionals?”, *Mind*, **83**, pp.278–81.

Corcoran, J.: 2003, “Aristotle's Prior Analytics and Boole's Laws of Thought”, *History and Philosophy of Logic*, **24**, 261–288.

Correia, M.: 2012, “Boethius on the square of opposition” in (Beziau and Jacquettei 2012), pp.41-52.

Correia, M.: 2016, “The proto-exposition of Aristotelian categorical logic” in (Beziau and Basti 2016).

Costa, N.C.A. da: 1980, *Ensaio sobre os fundamentos da lógica,* HUCITEC, São Paulo.

Costa, N.C.A. da & J.-Y.Beziau: 1994, "Théorie de la valuation", *Logique et Analyse*, **146**, pp.95-117.

Costa, N.C.A. da, J.-Y.Beziau & O.A.S.Bueno: 1995, "Paraconsistent logic in a historical perspective", *Logique et Analyse*, **150-152**, pp.111-125.

Dieudonné, J. : 1987, *Pour l'honneur de l'esprit humain*, Hachette, Paris. (English translation : *Mathematics – The music of reason*, Springer, Berlin, 1992).

D'Ottaviano, I. M. L. & da Costa, N. C. A.; 1970, "Sur un problème de Jaśkowski », *Comptes Rendus de l'Académie des Sciences de Paris*, **270**, pp. 1349–1353.

Geach, P.: 1950, "Subject and Predicate", *Mind*, **59**, pp. 461-482.

Gomes, E.L and I.M.L.D'Ottaviano.: 2010, "Aristotle's theory of deduction and paraconsistency", *Principia* **14***,* pp.71-98.

Granger, G.-G.: 1976, *La théorie aristotélicienne de la science*, Aubier, Paris.

Gourinat, J.-B.: 2009, "Le traité de *De l'interprétation* entre logique classique et logique non-classique", in S.Husson (ed), Interpréter le De interpretatione, Paris, Vrin, pp.164-192.

Gourinat, J.-B.: 2010, *La dialectique des stoïciens*, Paris, Vrin.

Heijenoort, J. van.: 1974, "Subject and Predicate in Western Logic", *Philosophy East and West*, **24**, pp. 253-268.

Hertz, P.: 1931, "Von Wesen des Logischen, insbesondere der Bedeutung des modus Barbara", *Erkenntnis* 2, 369–392.

Hilbert, D.: 1899 *Grundlagen der Geometrie*, Leipzig: Teubner (English translation: *Foundations of Geometry*).

Hintikka, J.: 2012, "Which mathematical logic is the logic of mathematics?, *Logica Universalis*, 6, pp.459-475.

Horn, L.: 2015, "On the contrary: disjunctive syllogism and pragmatic strengthening", in A.Koslow and A.Buchsbaum (eds), *The Road to Universal Logic - Festschrift for 50th Birthday of Jean-Yves Béziau Volume I*, Birkhäuser, Basel, 2015, pp.241-265.

Jones, R.E.: 2010, "Truth and contradiction in Aristotle's *De Interpretatione* 6-9", *Phronesis* 55, pp.26-67.

Kneale W. and Kneale M.: 1962, *The development of logic*, Clarendon, Oxford.

Korzybski, A.: 1933, *Science and sanity - An introduction to non-Aristotelian systems and general semantics*, Institute of general semantics, New York.

Lachance, G.: 2016, “Platonic Contrariety (enantia): Ancestor of the Aristotelian Notion of Contradiction (antiphasis)?”, *Logica Universalis*, 10 (2016), pp.143-156.

Largeault, J. : 1993, *La Logique*, Presses universitaires de France, Paris, 1993.

Legris, J.: 2012, “Paul Hertz and the Origins of Structural Reasoning”, in (Beziau 2012a).

Lewis, C.I.: 1932, “Alternative systems of logic”, 42, pp.481-507.

Łukasiewicz, J.: 1910, *O zasadzie sprzecznosci u Arystotelesa. Studium krytyczne*, Kraków.

Łukasiewicz, J.: 1910, “Über den Satz von Widerspruch bei Aristoteles”, *Bulletin International de l’Académie des Sciences de Cracovíe: Classe de Philosophie*, p. 15-38.

Łukasiewicz, J.: 1920, “O logice trójwartosciowej”, *Ruch Filozoficny*, **5**, pp.170-171.

Łukasiewicz, J.: 1927, “O logice stoików”, *Przeglad Filozoficny*, **30**, pp.278-299.

Łukasiewicz, J.: 1951, *Aristotle's syllogistic: from the standpoint of modern formal logic*, Oxford University Press, Oxford.

Łukasiewicz, J.: 1953, “A system of modal logic”, *Journal of Computing Systems*, **1**, pp. 111-149.

Markin, V.I.: 2013, “What trends in non-classical logic were anticipated by Nikolai Vasiliev?”, *Logical Investigations*, **19**.

Moisil, G.: 1972, *Essais sur les logiques non-Chrysipiennes*, Académie de la République Socialiste de Roumanie, Bucarest.

Murinová, P. and Novák, V.: 2016, “Syllogisms and 5-Square of Opposition with Intermediate Quantifiers in Fuzzy Natural Logic”, *Logica Universalis*, 10 (2016), pp.339-357.

Patterson, R. : 1995, *Aristotle’s modal logic*, Cambridge University Press, Cambridge.

Priest, G.: 1979, “The logic of paradox”, *Journal of Philosophical Logic*, **8**, pp.219-241.

Raspa, V.: 1999, “Łukasiewicz on the Principle of Contradiction”, *Journal of Philosophical Research* , 24, pp. 57-112.

Reiser, O.: 1935, “Non-Aristotelian logics”, 45, pp.110-117.

Resnik, M.: 1973/4, “The Frege-Hilbert Controversy,” *Philosophy and Phenomenological Research*, **34**, pp.386–403.

Rini, A. : 2011, *Aristotle’s modal proofs*, Springer, Dordrecht.

Rooij, van R. : 2012, “The propositional and relational syllogistic”, *Logique et Analyse.*

Seddon, F.: 1996, *Aristotle & Łukasiewicz on the Principle of Contradiction*, Modern Logic Publishing, Ames.

Scholz, H.:, 1931, *Abriss der Geschichte der Logik*, Karl Alber, Freiburg.

Szabó, A.: 1969, *Anfänge der griechischen Mathematik*, Akademiai Kiádo, Budapest (English translation: *The Beginnings of Greek mathematics*, Kluwer, Dordrecht, 1978).

Slater, B. H.: 1995, “Paraconsistent logics?”, *Journal of Philosophical Logic* **24**, pp.451–154.

Smirnov, V.A.: 1989, “The logical ideas of N.A.Vasiliev and modern logic”, In: *Logic, Methodology and Philosophy of Science* (Studies in Logic and Foundations of Mathematics). Vol. 126. Amsterdam. pp. 625 – 640.

Smith, H.B.: 1918, “Non-Aristotelian logic”, *The Journal of Philosophy, Psychology and Scientific Methods*, **15**, pp. 453-458,

Smith, R.: 1984, “Aristotle as proof theorist”, *Philosophia Naturalis*, 21, 590-597.

Suchon, W.: 1999, "Vasiliev: what did he exactly do?", *Logic and Logical Philosophy*, **7**, 131–141.

Tarski, A.: 1936, 0 *logice matematycznej i metodzie dedukcyjnej*, Ksiaznica-Atlas, Lwów and Warsaw (4th English edition by Jan Tarski: *Introduction to Logic and to the Methodology of the Deductive Sciences*, OUP, Oxford, 1994).

Tarski, A.: 1937, “Sur la méthode déductive”, in *Travaux du IXe Congrès International de Philosophie*, VI, Hermann, Paris, pp.95-103.

Vandoulakis, M. :, 1998, “Was Euclid's Approach ta Arithmetic Axiomatic ?” in *Oriens Occidens* , 2, pp.141-181.

Vasiliev, N.A.: 1910, *On partial judgments, triangle of opposition, law of excluded forth*, Kazan.

Vasiliev, N.A.: 1912, “Imaginary (non-Aristotelian) logic”, *The Journal of the Ministry of Education*, 40, pp. 207-246, English translation in *Logique et Analyse*, 182 (2003), pp.127-163.

Vasiliev, N.A.: 1913, “Logic and metalogic”, Logos 1/2, pp.53-81, English translation in *Axiomathes*, 3 (1993), pp.329-351.

Westerståhl, D.: 1989, “Aristotelian Syllogisms and Generalized Quantifiers” *Studia Logica*, 48, pp. 577-585.

Vogt, A.E. van : 1945, “The world of null A”, *Astounding stories*.

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