There is no cube of opposition

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Abstract. The theory of opposition has been famously crystallized in a square. One of the most common generalizations of the square is a cube of opposition. We show here that there is no cube such that each of its faces is a square of opposition. We discuss the question of generalization and present two other generalizations of the theory of opposition to the third dimension: one based on Blanché's hexagon of opposition, the other on the square of contrariety.

Mathematics Subject Classification (2000). Primary 03A05; Secondary 00A30; 03B45, 03B53, 03B22, 03B50.

Keywords. Square of Opposition, Cube of Opposition, Hexagon of Opposition, n-opposition, Generalization.

1. The cube of opposition: an obvious geometrical generalization

An obvious way to generalize the square of opposition is to consider a cube of opposition. Many cubes of opposition have been presented in the literature (see e.g. [39], [50], [51], [19], [40], [29], [27], [30], [31]). The cube is a an immediate generalization that one may have for the theory of opposition driven by a geometrical spirit. From a square we can go to other polygons: a pentagon, a hexagon, a heptagon, ..., a chiliagon. And such generalizations also exist in the literature (see e.g. [36], [26], [37] and in general all the recent publications on the square: [12], [14], [15], [16], [17], [18]).

The cube is a nice generalization in the sense that we keep the square shape but at the same time we go to the third dimension. Something is preserved and at the same time there is a change, an expansion. This double contrasting aspect - preservation with transformation - is a key feature of generalization. But this is here only from the geometrical point of view.

Another fundamental aspect should be taken into account, and that it is: the relation between the theory and what it is supposed to represent. There is an interaction between the geometrical figure of the square and the theory of opposition and this interaction also has to be preserved. Generalization should not just be on the geometrical side, it should also be on the side of what the geometrical object is supposed to represent. This side is not a dark side and only one side. The square has internal and external structures that can be colourfully represented. The square can be seen as built on a red cross (the heart of the square), which is then "circled" by a top blue line of contrariety, a bottom green line of subcontrariety and two black arrows of subalternation (Fig 1).[1]

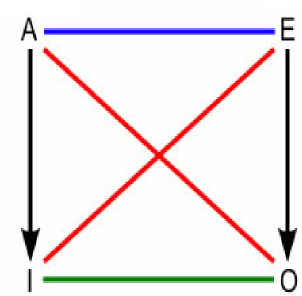

Figure 1 - Abstract coloured square of opposition

The result presented in this paper shows that there is no straightforward generalization of the theory of oppositions from a square to a cube, in the sense that there is no cube of opposition such that each of its six faces is a square of opposition as represented in Fig 1.

The title of our paper is deliberately provocative. It is to stress that if one wants to promote the idea of a cube of opposition, (s)he has to carefully explain and/or justify what (s)he is doing. We will let the proposers of such cubes of opposition do the job. Here we will present two other generalizations of the square into the third dimension which are not cubes, and explain why they are good generalizations of the square of opposition.

2. The square of opposition: a flag for the theory of opposition

The theory of oppositions has famously been crystallized in a square. This crystallization became very important, exceeding the theory itself. It is not exaggerated to say that the square of opposition became the flag of the theory of opposition. And this is not necessarily a problem, this is quite a nice flag. This is indeed a better flag than most of countries' flags where there is no visible connection between the image and what it is supposed to represent. Let us examine two cases shown in Fig. 2.

[1]We introduced this colouring of the square in 2003, cf. [2].

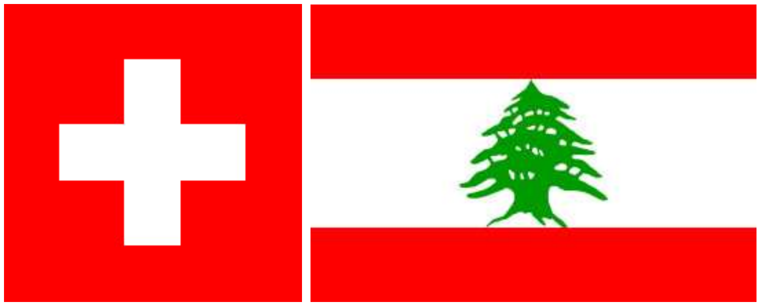

Figure 2 - Flags of Switzerland and Lebanon

The flag of Switzerland is a white cross on a red square. Like the square of opposition, it is a square, but here the square apparently has no special meaning. It is a pure question of regularity / symmetry in harmony with the cross which is inside. At the end everything is square in the Swiss flag. This can be seen as a compass indicating rationality and organization. Generally flags are rectangular. Only two sovereign states have a flag with a shape of a square: Switzerland and Vatican. We have organized congresses on the square in both of these countries (The 1st SQUARE in Montreux in 2007 and the 4th SQUARE at the Pontifical Lateran University in 2014). The others were organized in countries with rectangular flags. This is the case of the 3rd SQUARE that took place at the American University of Beirut in 2012. The flag of Lebanon is also made of some red and white geometric shapes but it has moreover at the middle of it a tree, known as the Lebanon cedar. This tree is the symbol of Lebanon, because it is *typical* of Lebanon. The square of opposition is also a *typical exemplification* of the theory of opposition. However such kind of "typicity" is not the same as the one of the Lebanon cedar.

Figure 3 - A typical representation of a giraffe

It is also not the same as the picture of a giraffe as represented in Fig. 3. Such a picture is a schematic representation of a giraffe emphasizing its main features corresponding to the standard definition of this animal: a long-necked, spotted quadruped ruminant. The class of giraffes is a class of homogeneous "things", so it is easier to "typify" them. What about the class of all animals? The giraffe in Fig. 3 certainly is not a good typical example of animal. It is not *general* enough. If we exemplify the notion of animal through this picture, this may give the idea that all animals are quadruped. Choosing the square represented as in Fig.1 this may also give the idea that opposition is necessarily a quadruped ... If we generalize the square of opposition to a cube or a hypercube, maybe the square may still serve as a good example, considering it is the first and simplest form. It would be the same as to consider 1 as a typical example of number.

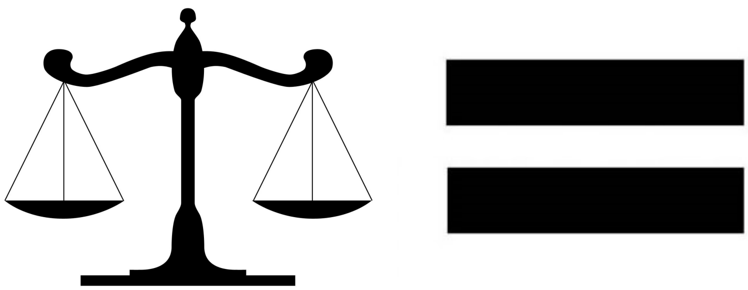

Figure 4 - Symbols of Justice and Equality

Can we say that the square is a *symbol* for the theory of opposition like the balance for justice, or the two parallel lines for identity (Fig. 4)? A symbol can be defined as a sign where there there is a connection between the sign and what it represents, as opposed to arbitrary signs (cf. Saussure [52] and [7] for a semiotic hexagon). The two signs of Fig. 4 are doubly symbolic: (1) They are stylized pictures ; (2) They represent an idea through a typical concrete example. We call the first aspect of symbolization *pictogrammatic symbolization* and the second aspect *ideal symbolization*.[2]

The picture of the giraffe presented in Fig. 3 can be considered as doubly symbolic. But there are two slight differences on both sides of the symbolization procedure. One the one hand it is not completely stylized–not so simplified both in form and colour, on the other hand it is not so ideally symbolic since the reality it describes, the species of giraffes, is not so ideal. The square of opposition presented in Fig. 3 is more stylized, despite the fact there are still colours. But colours are used here in different way than in the case of the giraffe. The colours themselves are symbolic like in the case of traffic signs; in the case of the giraffe the colours

[2]We have elaborated this distinction in our paper "La puissance du symbole" [9] published in the book *La pointure du Symbole* [13] which is the result of the interdisciplinary workshop we organized at the University of Neuchâtel in 2005. Saussure gives as an example of symbol the balance but he does not specify the double aspect of symbolization.

are purely descriptive.[3] Considering the ideal aspect of symbolization, the theory of opposition is much more ideal than the species of the giraffes. The question we have to investigate is if the figure of the square is as good an idealization through particularization as is the balance for justice or the two parallel lines for equality.

Since justice and identity are very heterogeneous, the objects singled out to represent them are necessarily too particular. The art of ideal symbolization is to convey the general idea through a particular concrete instantiation. The problem is that with the theory of opposition we are going from the particular to the general, whereas this is not the case with justice, as the theory of justice did not start with a balance. Going from the particular to the general is very common in science, in particular in mathematics (see e.g. [33].

Fig. 1 can be viewed as an abstract structure having many different instantiations. The letters A, E, I, O can be seen as variables that can be interpreted as different kinds of propositions or different kinds of concepts. Let us just give two examples: the square of modalities and the square of speed (Fig. 5). The square of modalities can be interpreted as a square of concepts (necessary, possible, impossible, not necessary) or of correlated propositions (It is necessary that it will rain, etc ...). These modalities can also be interpreted in a deontic way (obligatory, prohibited, ...) of which the square of speed is a particular case related to action.

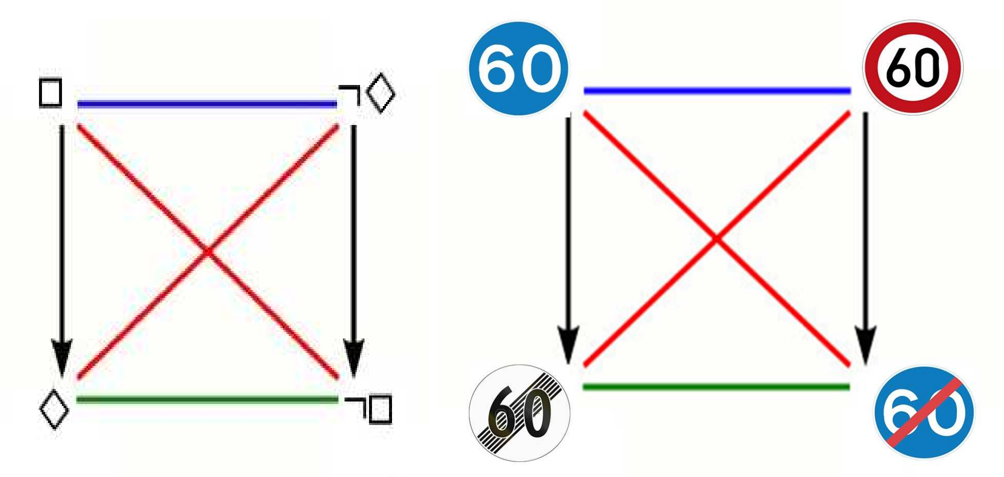

Figure 5 - Square of Modality and Square of Speed

[3]Our choice for the colours of the square as in Fig. 1 was more or less intuitive: red for contradiction, because it is the strongest opposition, black for subalternation, because it is not an opposition. The choice of blue and green was more intuitive, we didn't know at this time the RBG theory which was later on formalized by Dany Jaspers using the theory of opposition, see [35].

Historically speaking the situation developed the other way round. First a particular square was developed, a square related to Aristotle's theory of propositions, which classifies the propositions in four categories.[4] There are here already two levels: the categories themselves (universal affirmative, universal negative, particular affirmative and particular negative) and specific examples. In Fig.6 on the left we have the original "typical" example given by Apuleius, the voluptuous square. It is very easy to understand through this particular example the corresponding categorical generalization, which is on the right.

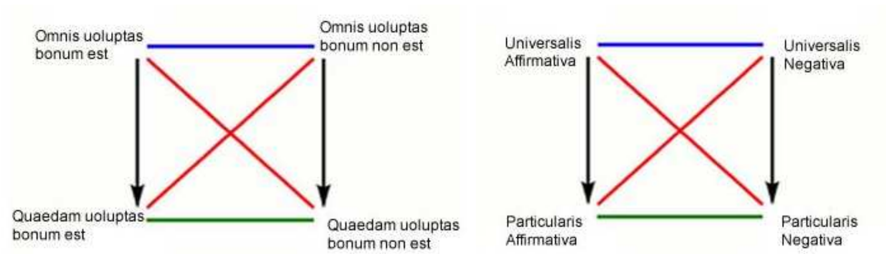

Figure 6 - The Voluptuous Square of Apuleius and the corresponding categorisation

People have generally not stuck to the original exemplification of Apuleius or/and to the Aristotelian categorization, but many have stuck to the square (and two of its avatars: the quantificational and modal squares) as if the theory of opposition was limited and/or reducible to that. Sticking to the original square is the same as to stick to natural numbers, not considering other numbers. But generalization in mathematics is not the product of pure fantasy. Irrational numbers are the by-product of rationality, more specifically the reduction to the absurd.

Aristotle's theory of propositions led to a specific configuration of the theory of opposition. By abstraction a certain structure is manifested and then applied back to many particular cases. This procedure is common in mathematics where structures like algebraic structures were extracted from some specific cases, studied by themselves and applied back to some concretes cases.[5] Two famous cases are groups and lattices. Some people even had the funny idea that everything is (or has the structure of) a group. Other people had a similar idea about lattices. In fact at some point lattices were called "structures", as if they were the quintessence of structures (see [32]). But the idea of structures was indeed the next step in generalization by abstraction in mathematics.

Saying that the theory of opposition is nothing more than the square of opposition would be the same as saying that geometry is nothing more than Euclidean geometry or that numbers are nothing more than natural numbers. Nevertheless

[4]The square of opposition is an interesting way to classify propositions and it can be seen as a tool for classification, which is at once more complex yet more compact than the most famous classificatory structure, the tree–about the theory of cllasification see [47].

[5]Let us point out here that there is a difference between generalization reached by induction and generalization reached by abstraction from a single example. There can be some mixed cases. In the case of the square it looks more like pure abstraction than induction.

we can use the square of opposition as a flag for the theory of opposition because it was the first manifestation of it. This is a phenomenon common in thought and language. “Alpinism” means mountain climbing, not only climbing the Alps. Some people are trying to detach it from its particularism and replace it by “mountaineering”. Another possibility would be to talk about “Everestism”, considering that Mount Everest is the highest mountain on earth (the name of this mountain is related to George Everest, the uncle of Mary Everest Boole, the wife of George Boole). Using proper names, another option would be “Saussurism”, in memory of Horace-Bénédict de Saussure, one of the main promoters of Alpinism. For the square it is also common to attach it to Aristotle, Apuleius or Boethius. When one is talking about the Apuleian square, we know it is about opposition, not just about a geometrical shape or/and Apuleius. This conveys the idea of the theory of opposition.

Saussure was not the first to climb the Mount Blanc, nor George Everest was the first to climb the Mount Everest. And probably Apuleius is not the first to have drawn a square of opposition (see [28]), as it has been claimed by Bocheński [24] [25] and Sullyvan [54]and supported by Londey and Johanson:

> Historians of logic are agreed that, although Aristotle stated the principal logical relations between the four types of categorical proposition, he did not invent the heuristic diagram, traditionally known as the Square of Opposition, which maps those relations. This diagram has been part of the staple fare of students of elementary logic for centuries, but modern writers do not always show any certainty about its origin, or its original form. It is not uncommonly thought to be a medieval invention, or is simply glossed as 'traditional' in a way which implies either a medieval or post-medieval origin. However, Bocheński and Sullyvan correctly locate the first known occurrence of the diagram in the *Peri Heremeneias*. “The Apuleian square of opposition”, Appendix B of [41], p.108,

Let us point out that nobody has seen a square of opposition drawn by Apuleius though Londey and Johanson correctly say that Apuleius gives a “set of instructions on how to draw the figure and how to label the relations to be charted”. But Laurence Horn pointed out that Aristotle also had a square in mind (see [34]).

3. The proof that there is no cube of opposition

We now present the proof that there is no cube of opposition. Firstly we present an abstract proof and secondly a visual proof.

THEOREM *There is no cube of opposition such that each side of it is a square of opposition.*

Abstract proof. 1. Suppose that we have a cube of opposition such that each of the six faces of it is a standard square of opposition. 2. At a vertex v of a cube we have a triple point where three edges A, B, C coterminate and three faces X, Y, Z meet. 3. Any pair of these three faces share one of these three edges, and any pair of these three edges form two adjoining sides of one of these three faces. 4. According to the definition of a square of opposition, when we have two edges meeting at a corner of a square, one should be black (subalternation) and one should not be black (either green or blue). 5. Therefore one of the edges meeting at v must be black, let's say A. 6. If B is black too, then, according to (3), A and B are two edges of a square, say X, meeting at a corner of this square, so X is not a square of opposition, this contradicts (1). 7. So B is not black. 8. Then according to (4) C has to be black. 9. According to (3) B and C meet at a corner of one of the squares, say Y. 10. Since B is not black, according to (4), C must be black, 11. But then A and C are two black edges meeting at a corner of the third square Z, so Z is not a square of opposition, this contradicts (1).

Visual proof. There is a more visual and more direct way to prove this result.[6] Consider the following situation.

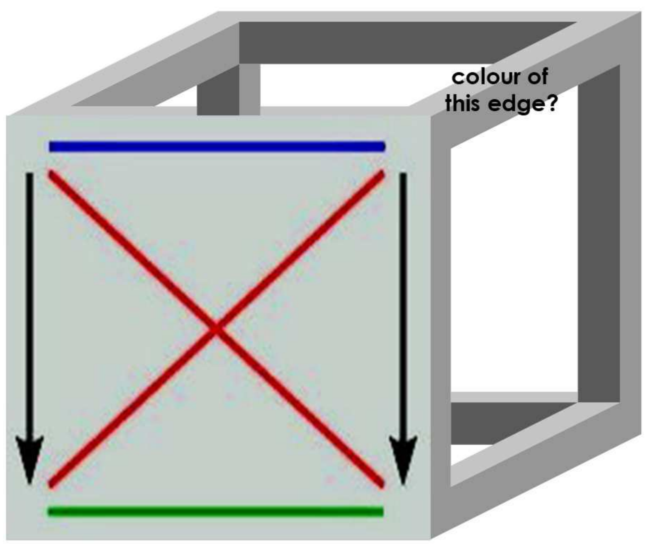

Figure 7. No Cube of Opposition

We have put a square, in the standard position, on the front side of the cube. It is easy to understand that there is no loss of generality putting the square in this position.

Now let us consider the diagonal edge on the top right. It can be blue, green or black. Blue: then the upper side of the cube is not a square of opposition Green:

[6]About recent advances on visual reasoning see e.g. [43]).

then again the upper side of the cube is not a square of opposition Black: then the right side of the cube is not a square of opposition

In the three cases, due to colouring, we immediately see why the mentioned sides are not squares of opposition even without stating explicitly the above proposition (4).

4. Two other three-dimensional generalizations of the square of opposition

In this section we will discuss two other generalizations of the square of opposition: the hexagon of opposition and n-opposition theory. They are also related to the third dimension but in a different way than the cube. In a way which is at the same time more indirect and more fundamental. These two generalizations have in common the fact that the first motivation of their development is not the third dimension, but they naturally and even imperatively lead to it. They are also tightly related to each other, n-opposition theory being a generalization of the hexagon of opposition.

The hexagon of opposition appeared in the 1950s, and as often in the evolution of science, which can be seen as a general movement of human thought, it is not the idea of one isolated person. Different people had independently the same idea at more or less the same time. Let us note that this is what happened with many-valued logic which was independently developed by Peirce, Post, Bernays, and Łukasiewicz. This does not mean that everybody had exactly the same idea. There is something in common, but it is presented and developed in different ways, and this can lead to a theory which is a blend of ideas, or alternatively one of them develops more than the other ones and dominates.

In the case of the hexagon, this is rather the second case, as Robert Blanché developed the hexagon of opposition in a systematic and continuous way over more than 10 years starting in the mid 1950s (see [20], [21], [22], [23]). We have already written a paper on the hexagon entitled "The Power of the Hexagon" (see [12] and edited a special issue of *Logica Universalis* on the hexagon (see [12]) so we will not enter much into details here. We will just discuss the hexagon from the problematic of generalization.

Blanché's hexagon of opposition is not a two-dimensional generalization of the square of opposition in the sense that sides are added. Let us point out that the square of opposition is not just a square, it has a structure made of three oppositions and the further notion of subalternation. If we add sides, how to adjust the structure and what is the motivation?

Blanché's construction is based on a true philosophical inquiry about the theory of opposition dealing in particular with the original exemplification of the square, the square of quantification. Blanché solved one of the main problems of the square of quantification. There were two problems with the traditional square, both related with the I-corner, the "existential" corner. The first problem is the question of *existential import* that we will not discuss here (a very romantic topic which bears some similarity with the question of *sex of angels*). The second problem is about *some*. It was pointed out by several people that the I-corner does not correspond to the meaning of the quantifier some and for this reason people wanted to replace the square by a triangle (Fig. 8).

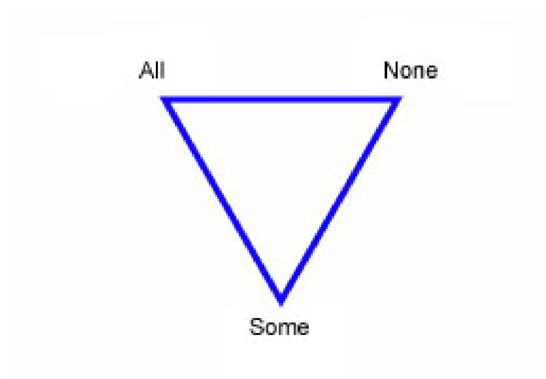

Figure 8. Quantificational Triangle of Contrariety

Blanché, instead of staying with just the triangle, constructed a hexagon through a star by tying this triangle of contrariety with a triangle of subcontrariety (Fig. 9).

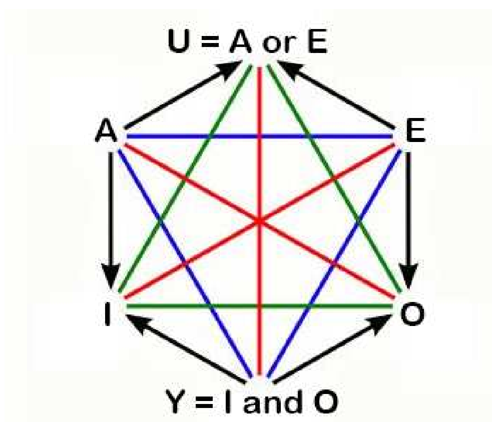

Figure 9. Blanché's Hexagon of Opposition

In which sense can we say that this hexagon of opposition is a generalization of the square? Firstly in the sense that the hexagon has a high degree of generalization, it can be applied to many different situations. But this is also the case of the blue triangle which is the heart of hexagon. Secondly it is a natural *extension* of the square, which is recovered inside the hexagon (and two more squares appear as can be seen by rotating the hexagon). It is natural to consider the conjunction of the I and the O corners and the disjunction of the A and E corners. This makes sense, and it is supported by a nice internal structure. So the hexagon is like a flourishing of the square. It is a natural *complexification* of the square. Such kinds of developments contrast with trivial generalizations.

The way from the hexagon to the third dimension is also a kind of flourishing. Having discovered that the negation of necessity was a paraconsistent negation (see [1], [4], [5]) I wanted to systematically study the relations between negation and modalities (see [3], [6]). This can lead to an octagon. It is one option, but I wanted to preserve the star / hexagon structure, therefore I built three hexagons of opposition and a natural way of relating them is to construct a three-dimensional object. So I built such a structure. As noted by Alessio Moretti and Hans Smessaert to whom I communicated my idea at this time a fourth hexagon shows up in this three-dimensional construction, that I first saw as a stellar dodecahedron but that they identified as a cuboctahedron considering the surfaces generated by subalternation (FIg. 10). The surface of this cuboctahedron is made of an alternation of 6 squares with 8 triangles. None of these squares is a square of opposition, all the edges are subalternations, that's why they are in black in Fig. 10. In red we have links of contradiction. We have not put the blue and green edges of contrariety and subcontrariety but at the end we have 12 interlaced squares of opposition inside this three-dimensional object because we have 4 hexagons and inside a hexagon of opposition there are 3 squares of opposition.

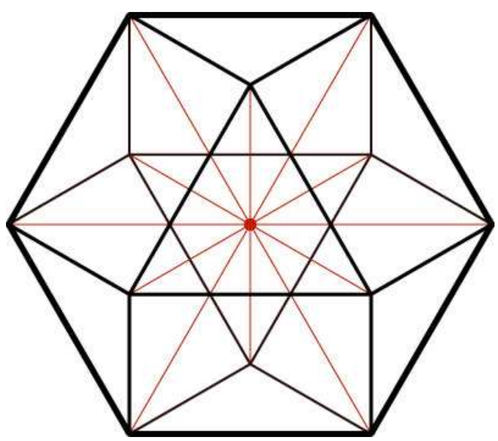

Figure 10. The Cuboctahedron - with a structure of 4 hexagons

Let us now have a look at the other generalization which also leads to the third dimension, n-opposition theory. As we have seen, the hexagon is constructed by putting two triangles together. The heart of the hexagon is the blue triangle of contrariety. Such a triangle can be seen as breaking / extending the dichotomy promoted by the school of Pythagoras (cf. the table of opposition). This triangle was not designed by Aristotle, but Aristotle promoted the notion of contrariety which is intimately related to it.

On the other hand contrariety is not necessarily limited to trichotomy. We can go to tetrachotomy and draw some squares of contrariety. The similarity of a square of contrariety and the standard square of opposition is only in the structure/shape of the edges, but all edges correspond to contrariety, like with the triangle of contrariety, so using colour, we have a blue square (Fig. 11).

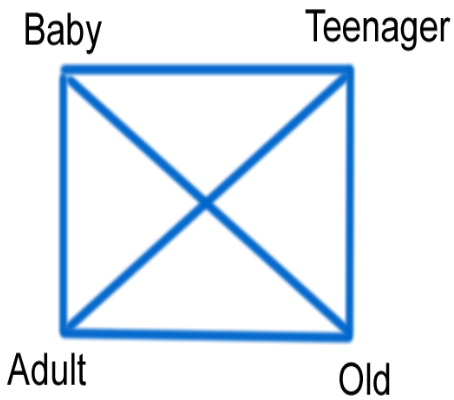

Figure 11. Square of Contrariety of Ages

In the same way as in the construction of the hexagon, we can consider a dual green square of subcontrariety and tie the two squares together using red contradictory edges. This gives birth to an octagon of opposition. This is a natural generalization of the hexagon. We have gone from a 3-contrariety (a contrariety of three terms) to a 4-contrariety (a contrariety of four terms). Contrariety can work with only 2 terms but it is not limited to 3 terms. We can go to 4 terms and more. In fact it seems that "child" is missing in the square of figure 11, so it would be better to have a pentagon.

I promoted this generalization of the theory of the hexagon of opposition and Alessio Moretti baptized this "n-opposition theory" (cf. [44]). The expression is quite ambiguous because the number of oppositions is still the same in all the cases, we are not adding more oppositions, we are staying only with the three basic ones: contariety, subcontrariety and contradiction. 3-opposition theory is Blanché's hexagon, 4-opposition theory is when we consider 4 contrarieties (and 4 subcontrarieties), etc. But the main contribution of Moretti is much more interesting than this ambiguous terminology (which he likes to abbreviate as N.O.T.= n-opposition theory).

Moretti had the idea that it would be better to have the same distance between the four vertices. This is not the case in a square because the diagonals are longer than the sides. For the standard square this is not necessarily a problem, because the diagonals do not correspond to the same notion of opposition as the sides. One may defend the idea that contradiction is longer because it is stronger. But if we consider a blue square of contrariety then the asymmetry is a defect. So Moretti suggested that it would be better to consider a tetrahedron (Fig. 12). Such a geometrical object is three-dimensional. We go here to the third-dimension by a kind of accidental necessity.

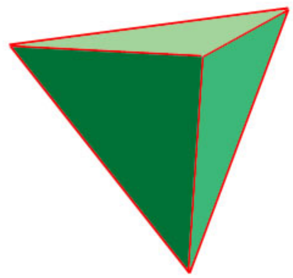

Figure 12. Tetrahedron

If we want now to generalize the construction of the hexagon for 4-opposition, we construct a dual green tetrahedron of subcontrariety and putting the two together we arrive at the object represented in Fig. 13, which is called a "stellated octahedron".

Figure 13. Stellated Octahedron for 4-opposition Theory

The idea of tetrahedron can be generalized to any number of vertices, the name for such an object is "simplex" and the name for a composition of two simplexes is a bi-simplex. Moretti used these geometrical objects to generalize Blanché's hexagon of opposition. His theory of n-opposition takes a bi-simplicial form and even a poly-simplicial from (see [45], [46]).

Let us summarize the story. We have the square of opposition which is a two-dimensional object. Considering trichotomy is quite natural and does not lead immediately to the third dimension. It leads to a hexagon. On the one hand instead of going to an octagon where the basic figure of the triangle is somewhat lost, we can go to the third dimension constructing a cuboctahedron - all this staying in 3-opposition theory. On the other hand we can generalize this theory going to 4-opposition theory, then we have to go directly to the third dimension right at the start.

In the case of the cuboctahedron, the move to the third dimension is motivated by the preservation of a triangular structure. The triangle leads to the third dimension, this is quite homogeneous and harmonious. In the case of 4-opposition, although the way of going to the third dimension is more subtle than in the case of the cube of opposition (internal structural necessity) this theory can also be criticized. Why go from trichotomy to tetrachotomy? does this make sense from a philosophical point of view? The move from 2 to 3 can already be criticized, as Kant puts it: only dichotomy is *a priori*(see [38]). But trichotomy can indeed be defended, either from the point of view of reality or from a transcendental viewpoint, or both. The structure of thought can be seen as trichotomic. This is an idea more or less promoted by Blanché, justifying his hexagon (see [23]). We can consider that at the level of signs everything can be reduced to dichotomy but that thought is essentially trichotomic. Tetrachotomy looks much more empirical. There are four seasons (in some regions of the earth), but can we say that there are four kinds of ages? As we have said, five would be a better division.

The four points of the compass are a rather arbitrary squaring of space. Maybe everything is round, in space and time (Fig. 14).

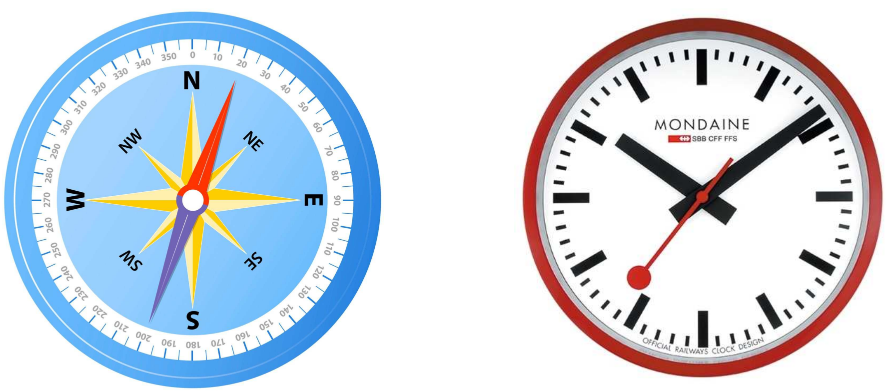

Figure 14 - Circles of Space and Time

Acknowledgements

Thanks to Catherine Chantilly and Robert Purdy for discussion and comments.

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