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ABSTRACT. In a first part we discuss what can be considered as a game or not, examining some typical games, giving a formula characterizing games, differentiating games from science, art, religion, sport and business. In a second part we discuss a central game notion, the notion of rule. We distinguish four categories of rules for a game: framework rules, deontic rules, strategic rules and teleological rules. Our discussion is illustrated by many examples ranging from chess to soccer via frescobol and doll's tea party.

> Dedicated to Walter Carnielli. 60: the game is just starting!

We are, above all, rule-making and rule-following creatures. It follows immediately both that the philosophical clarification of the concept of a rule is a suitable large topic to engage philosophers and that any radically new insight about rules will have substantial repercussion on a wide range of philosophical reflections. ([Baker and Hacker, 1985], pp.56–67)

0. Start

The notion of game has been increasingly popular. But what is a game? In this paper we intend to deal with the notion of game taken in a wide and wild sense, not limited to game theory or the fashionable game semantics, trying to characterize it through deep and essential features.

Most logicians or/and mathematicians who are using the word "game" are using it only metaphorically, using abstract symbols such as "players" or "winning strategy". They are not developing a philosophical analysis of what a game is, or presenting a theory of what a game is or should be. Game theory is not a theory of games, it is a theory roughly inspired by games. Game semantics also is not the semantics of games but a semantics inspired by games.

The relation between game and logic is generally only a one-way road: logical tools inspired by games. In this paper we will work in the other direction, applying deontic logic to games. This is very illuminating to understand for example a difference between chess and football besides the physical/intellectual duality: as we will explain, chess is a allowance game by contract to football, a prohibition game.

There are books analyzing a specific game such as chess (e.g. [Lasker, 1950]), books about the sociology/ethnology of games (e.g. [Wendling, 2002]), but no books about the general nature of a game. The heterogeneity and many dimensions of games may explain this situation. The only exception is the excellent book by Roger Caillois (1913-1978), Les jeux et les hommes [1957]. Although this book has been translated in English (Man, play and games) it is not very well known, especially among philosophers. Our paper can be seen as an improvement of Caillois's analysis, but it does not presuppose the knowledge of this book (we advise nevertheless anybody interested in "game theory" or "ludology" to read this book). A central difference between our approach and Caillois's is the way that we are considering rules. The main subject of our paper is in fact beyond the notion of game, it is the notion of rule, but what we are dealing with are not rules in the air, they are rules of the game. Our paper is neither argumentative, nor an overview of previous works, philosophy does not reduce to this dichotomy, there is also conceptual analysis. An important objective of philosophy is clarification and understanding. Many philosophers have worked in this direction, recently Ludwig Wittgenstein. The central notion of our paper, the notion of rule, has been much discussed by Wittgenstein, but although we have been inspired by him, the present paper is not a comment about his work.

1 Game: what it is and what it is not

Maybe the world and/or the universe is a game (cf. [Israel and Gasca, 2009]). But before considering such a brainy issue, let us rather consider the universe of games. There are plenty of games, very different from each other. We can start with an eagle eye, a panoramic view of what are things called games and then try to classify and structure the universe of games.

A purely descriptive approach to games, as to many notions, would be chaotic, and classification is only a first step for developing a better understanding. The idea is to go further on by building a normative concept of game excluding on the one hand things that are usually considered as games and on the other hand including things that are not usually called games. But a too normative concept of game would be absurd as well as a trivial concept of game, according to which everything is a game. The notion of game can be illuminating but we must be careful not to be dazzled.

1.1 Four basic games

To start let us fix our ideas with four kinds of game we have carefully chosen to be representative of the different features of a game: *football, chess, bilboquet* and *monopoly*. The reason why we have chosen these games will clearly appear to the reader when we will analyze the main characteristic of what a game is. Moreover these games are more or less universal. Our study has been conducted by consulting encyclopedic books about games (e.g. [Encyclopedia of Games, 1998], visiting museums such as the beautiful Swiss Museum of Games (located in a Castle at La Tour de Peliz by Lake Geneva) and last but not least practicing games.

Despite the world cup, football is not very popular in the USA and Americans are using a different name for it ("soccer"). Football is from British origin, but the most popular footballing country is since many years Brazil, in the same way we can say that Russia is the kingdom of chess, monopoly typically American in spirit and bilboquet a frenchy entertainment, *intraduisible!* Most

of the games can be classified into these four categories of game, or considered as a mixture of these categories. Let us briefly describe these games.



Football is a game based on physical activity opposing 11 against 11 players on a field, but also with additional actors of the game inside and outside such as referees and coaches, 90 minutes is the end of the game. A similar game is tennis, with only 2 players on a physical space called court, it is potentially unlimited in time, the end of the game is a score. A variant of it is called *frescobol* in Brazil (much popular in Copacabana), and *matkot* in Israel, where it is considered as the country's national sport. It is played on the beach with two players, with no loser or winner, the end of the game is when the ball disappears in the sand or on the sea.



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Chess is a game on a board confronting 16 pieces against 16 pieces directed by 2 intelligent players (natural or artificial), checkmate is the end of the game, chance and physical skills are not part of the game.

Monopoly is a racing board games son of the *game of the goose*, the race does not reduce to chance, financial skill is essential, end of the game is bankruptcy. A variation of it is *hopscotch*, where ability is physical. Another variation is *game of life*, a Bradley game, updated version of Milton Bradley's original game: you start the game deciding whether jump ahead on the game board or pay your way into the college. Going to the college gets you the advantage of 3 shots at a good salary instead of 1 but will set you back financially to start. Along the road of life, you will get married, choose what kind of house to live in, buy insurance, and ride the ups and downs on the road of life. This Bradley game should not to be confused with Conways game of life, name related here to the biological sense of life. This is a zero player game: once the "pieces" are placed in the starting position, the rules determine everything that happens later. Nevertheless, Life is full of surprises! In most cases, it is impossible to look at a starting position (or pattern) and see what will happen in the future. The only way to find out is to follow the rules of the game.



Bilboquet was promoted by the king of France Henri III in the 16th century, he liked to play bilboquet to impress his mignons. There is a handle, a string and a ball with a hole. There is only one player winning when getting the (hole of the) ball on the handle. A variation of this game is much popular in Japan under the name *Kendama*. *Pinball* can be also seen as a variant of bilboquet, as well as the ancestor of pinball, *billiards*, and also *golf*. In all these games, there is a pin, a ball and hole. The British have a female version of bilboquet called *cup and balls*.

1.2 The chemistry of game

To understand what a game is, it is important to be able to argue with good criteria what is not a game. To do so we can distinguish 5 features of a game: competition (Co), fiction (Fi), chance (Ch), skill (Sk) and fun (Fu). Our idea is that an activity is a game if and only if it obeys the following formula:

$$Co + Fi + (Ch \% Sk) + Fu.$$

Translating the chemistry of symbolism¹, this means that a game must be a competition, a fiction, that it involves either or both chance and skill and last

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¹We are using chemical metaphors in honour of C.S.Peirce, great chemist and logician

but not least, a game is fun. It is quite easy to see that the games we have been discussing obey this formula, although the *Co* feature is not always explicit. Brazilians and Israelis like to say that frescobol/matkot favors cooperation rather that rivalry, but nevertheless the winning strategy is to keep the ball in the air and competitions is won by rival duos, champion is the duo who was able to keep the ball for the longer time. Football and other team games favor also cooperation, the difference is that football cannot really be played with only one team.

Based on this chemistry of game it is not difficult to eliminate from the universe of games activities such as business, cooking, praying. astronomy and playing piano. Business is maybe fun, it involves chance and skill, this can easily be seen as a competition, but when Pan American World Airways (Pan Am) reached bankruptcy leaving on the ground thousands of employees, this was no fiction. Praying may also be fun, it involves a certain skill, but it is not clearly a competition (race to heaven?) and not completely fictional (Jesus Christ is not essentially a movie character). What we can say about food and music is that they can be turned into games: the winner being for example the one producing the biggest sausage or the one conducting Beethoven 9th symphony in the fastest way. These activities are not fundamentally games, but like many activities, such as sexuality (erotic games), or even civil rights (American gambling of the green card), they can be degenerated into games. It would also be childish to consider astronomy, or any other science, as a game. Childish because science is not basically a competition to won the Nobel prize. Childish also because our world is not a fiction in a galaxy of possible worlds. It is true however than children are playing games to develop their cognitive faculties. But at some point (the skill of) the games turn into reality.

Talking about children games, we may wonder if our chemical formula is not eliminating too much: can we say for example that dolls tea party obeys the above magic game formula? What is missing is the *Co* factor. To solve this difficulty we may want to eliminate the *Co* factor from our formula, but then open the door of the house of games to many activities connected with acting. And in fact actors are said to play a role. A way then to exclude actors, like many sport men or women, would be to say they are not in a complete fictional state, since they are making a living of their fictional activities. A game that is essentially a work is no game. But this is not the only reason. The fact that most arts and sports are not necessarily fun is a connected reason to reject them of the game universe. Zeus will forgive us to consider than many Olympic games are not games according to our formula: is it fun to run 42,195 km?

But let us come back to children, another way to avoid the exclusion of children games is to understand the Co factor in a broad sense, the competition consisting in learning something, in most of cases, by imitation. In this sense of competition, doll's tea party is a competition with one's self, like bilboquet, this competition can be cooperative like frescobol, in the case of a many players doll's tea party.

The *Co* factor is one of the basic differences between our chemistry of game and the one of the most important game theoretist, Roger Caillois. For Caillois an activity is a game with the following features:

- 1. fun: the activity is chosen for its light-hearted character
- 2. separate: it is circumscribed in time and place
- 3. uncertain: the outcome of the activity is unforeseeable
- 4. non-productive: participation does not accomplish anything useful
- 5. fictitious: it is accompanied by the awareness of a different reality
- 6. governed by rules: the activity has rules that are different from everyday life

Skill is also pathologically missing from Caillois's definition. On the other hand in our definition we didn't put uncertainty, we rather implicitly included it through "Chance or Skill". In our formula we also don't have something corresponding to (6), the reason is not that we are considering games without rules but that rules are so important that we are no putting them at the same level as the five elements of our chemical formula.

Up to now, we have considered only human activities. What about animals, clouds and all the rest? Can we not say that the cat is playing with the mouse, that by chance a stone fell into the head of a monkey, that the world is a big game?

If we consider that fiction is an essential feature of a game and that fiction is crucially human, then there are no non human games. But some animals are quite similar to us it seems they can play roles. And more generally we may think that we are part of nature and that even our most sophisticated behaviors have some similarities with what is going in the world. It is no absurd to think, without much anthropomorphism, of a non-human activity involving fiction, fun, competition, competition, skill, at least an animal activity. If we are talking about stones, this is maybe another story, isn't it?

A way to clarify this issue, the question of the (rational) animal exclusivity with respect to games, is to examine one the most fundamental aspects of games we have not yet talked about, the notion of rules. Human games are based on rules created by human beings and to play a game is to take in account these rules. Can we say that a cow, a stone or a cloud creates rules and behave according to these rules?

2 Rules

A game is regulated by rules but to say that any activity regulated by rules is a game would quite naturally lead to the truism *everything is a game*; Kant opens his book of logic [1800] by saying that "Everything in nature, in the inanimate as well as the animate world, happens according to rules, although we do not always know these rules".

Can we say that rules of human games are similar to rules of other human activities and also with rules of animals, vegetables, minerals, atomic activities?

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2.1 Rules and laws

In Greece, 2700 years ago, human beings created some rules, the basis of a state: a democracy, an oligarchy, a monarchy. These are called *laws*, and we have the legislators, those creating the laws, and the legislation, a set of laws. It is also usual to speak about laws of nature and the Greeks were the first to describe such *laws of nature*, it was the beginning of science.

Why using the same word, what is the similarity between laws of a state and laws of nature? One could argue that a God (or some Gods) created the laws of nature, we have a divine legislator and a divine legislation. But does the stone obey the laws of God in the same way that citizen K pays her taxes? Kelly may not pay her taxes, but can the stone not obey the laws of God or Nature? It seems that when we are speaking of laws of nature there is an absolute necessity.

A different perspective to establish the bridge would be to argue that laws of nature are like civil laws, product of human minds. But the laws of nature are not completely arbitrary or/and product of our free will. They are describing natural phenomena, they are considered faithful, in particular they permit to make prediction, however this does not mean that they are absolutely necessary. If a stone does not follow the laws of nature, we will not put it in jail, we may on the contrary put it in a museum, as an extraordinary phenomenon. We may also change the laws to keep the reluctant stone in our net of scientific laws. According modern physics, the notion of chance is so important, it is not clear that there are absolute laws of nature, i.e. absolute regularity (see e.g. [Bohm, 1961]).

In case of doubt and to avoid confusion, we may decide to call *rules*, the legislative laws regulating human societies, by contrast to the laws of nature. But can we say that these rules, the rules of a society, are similar to rules of a game and that a society is a game? We will examine in the next section how rules of a game work, but it seems than since the start we can see an important difference. In a game, one *must* obey the rules, if he does not obey the rule he is *out of the game*. One may try to sustain the similarity saying that on the one hand cheating is part of the game, and that on the other hand one who does not obey the rules of society goes out of the society game, this outside being jail. Nevertheless it seems that in a game we are more like a rolling stone, mechanically and diligently following the rules. Rules in this sense look more imperative than laws of society. It is interesting to note that rules of chess and rules of football are officially qualified as laws, maybe to reinforce their imperative character (see Fide [2008] and Fifa [2010]).

Boole is famous for his book about the *laws* of thought [1854]. At first sight such laws look more like laws of nature. Our thoughts can be considered as part of nature, a position that could be reinforced by a neuroscientist establishing a relation between thought and brain activity. The wife of Boole, Mary Everest Boole, a gifted children teacher wrote many books, among them *Philosophy & Fun of Algebra*, where she says:

Arithmetic means dealing logically with facts which we know (about questions of number). 'Logically'; that is to say, in accordance with the 'Logos' or hidden wisdom, i.e. the laws of normal action of the human mind And this law of the Logos is made not by

any King or Parliament, but by whoever or whatever created the human mind governments have grown wiser by experience and found out that, as far as arithmetic goes, there is no use in ordering people to go contrary to the laws of the Logos, because the Logos has the whip hand, and knows its own business, and is master of the situation. ([Boole, 1891], p.1).

But can we not decide, independently of any Parliament or God, according to which rules we will reason, in the same way that we decide according to which rules we will play chess? Is mathematics, often considered as the highest form of reasoning, not a game for which we can set the rules? Our analysis of rules of a game will hopefully clarify these questions about laws of nature, laws of society, laws of thought².

2.2 Four kinds of rules

For any kind of game, we will distinguish four kinds of rules, presented in the following table:

Framework rules	setting the game	Where, when,
		with whom and with what
Deontic rules	defining the act of playing	What can be done or not,
		what have to be done or not
Strategic rules	qualifying the playing	How to play
Teleological rules	objective of the game	Why to play

TABLE 1 : FOUR KINDS OF RULE

In our characterization of rules, the less obvious category is the category of strategic rules, we will explain why they are fundamental and what is the relation between the other categories that we will also be described, showing how all these rules harmoniously work together³.

Framework rules

There is a tendency to directly connect rules with activities. But it is not absurd to consider rules in a wider sense, as conditions for activity. We can call rules the size of a football field, the duration of the game, the number of players, the sex of the players. We will qualify these rules as framework rules. Someone may think that this kind of rules are really different from a rule such as *one must not touch the ball with his hands*, because a football player, say Zidane, can obey or disobey (intentionally or not) this rule, but can he disobey a framework rules such as the size of the field? If he goes out of the field and goes on playing, we may say he is disobeying such rule. But there is a more radical way to disobey this framework rule, it is to play football within a field

²A special issue of *Logica Universalis* (vol.4, n2, 2010) on the theme "Is logic universal?" has recently been published discussing the question whether logic is eternal or not, including in particular a paper by Jaroslav Peregrin, "Logic and natural selection"

 $^{^{3}}$ Up to now, nobody has presented such analysis and classification of rules, for example Hintikka just presents a rough distinction between definitory and strategic rules (see e.g. [Hintikka, 2007], a dichotomy which confuses framework rules with deontic rules and which does not take into consideration the distinctive teleological rules, which are independent of deontic rules

of a size different from the official size. In this case the one who disobeys the rules (intentionally or not) is not an inside player like Zidane, but it can be someone in charge with the team business, say Bernard Tapie.

So with framework rules appear framework players. This is not really surprising, it is natural to say that the coach is part of the game, even if he is in some sense out of the game - he is outside of the field. In football there is also a strange inside player, this is the referee. In football the referee is only metaphorically inside the game, in fact he is outside the game, in the sense that he is not a competitor (contrarily to the coach). For him to be a competitor in the sense of playing in favor of a team is prohibited. This is the rule. The referee should be neutral. Also if the referee does not whistle the end of the match (intentionally or nor) at the right time, he is not obeying the rule (intentionally or not). So the referee also can obey or disobey rules, in particular framework rules.

Deontic rules

Deontic rules can be roughly characterized as what can be and cannot be done, have to be done and have not to be done, it is related with action once the framework is set, ruling the activity of the player within the game, although some framework rules can also be described by deontic modalities. We use the expression "deontic rules" because these rules can be described with the help of deontic modalities.

Deontic modalities can be explained with the following hexagon, which is an improvement of the square of deontic modalities, based on Blanchs ideas (see [Blanché, 1966], [Beziau and Payette, 2008], [Beziau and Payette, 2011], [Moretti, 2009]):



Optional: Allowed and Non-obligatory

In standard systems of deontic modalities, the six modalities can be defined starting with only one and defining all the other ones with classical negation, conjunction and disjunction. But our present example of deontic rules show how poorer this reductionist viewpoint is.

Football is a game dominated by prohibition rules, by contrast with chess dominated by allowance rules. In a reductionist viewpoint this would be the same in the sense that prohibited = not allowed and vice versa allowed = not prohibited. These identities do not reflect a strong difference of behaviors between acting applying rules of allowance and acting doing whatever is not prohibited.

The deontic rules of chess are rules of allowance: the move of the pieces is clearly defined, they explain what moves can be done. What is prohibited is not stated explicitly it is by definition the classical negation of all what is allowed. Moreover these rules are really rules of allowance, not excluding obligatory moves, sometimes there are no options left.

Football rules are on the contrary prohibition rules, they indicate what cannot be done. Most important and explicit prohibition rules are: not touching the ball with the hand (except for the goal-keeper), and the off-side (law 11 of Fifa, rightly called in French *hors jeu*). What can be done cannot be precisely defined: we cannot give rules for the move of football players, in principle all what is not prohibited is allowed but this is not exactly the case for example it is not prohibited for a player to lay down on the ground if he is not injured, but generally a player will not do so.

A game like monopoly is a obligatory-optional game, oscillating between rules of obligation and optional rules. There are rules specifying what has to be done: to pay taxes. But other rules stipulated things which are optional (allowed and not obligatory): to buy houses.

Teleological rules

We can say that teleological rules are connected with the question: why playing? There is an interesting ambiguity here. The simplest answer is: play to win. But then we may ask: why winning the game? This is related to: why to play? We will not in this paper directly answer this abyssal question.

In general a game has a beginning and an end. The end is connected with winning and/or losing. Winning is defined by actions which are neither prohibited, nor obligatory, according to the rules of the game, *optional actions* that the players are trying to perform (Frescobol, not as competition between dual team, can be seen in a dual perspective, nobody is losing or winning, but the players are trying to keep the ball in the air. We can describe the game by: it is obligatory to keep the ball in the air, if the ball falls in the sand or in the sea, it is the end of the game.)

A teleological rule is a rule that defines what these optional actions are. Teleological rules are at the same time independent and related to other rules. Check-mate in chess is a position in the chess board; it is a position ending the game. We can choose another end position for the game, not changing the framework and deontic rules. The choice of this teleological rule what is the winning position of the game is therefore independent of other rules. But at the same time the teleological rules is related with other rules: the winning action is something that should be neither too easy nor too difficult to perform following the framework and deontic rules.

The winning action should in any case be possible, for example we cannot choose as an end position in chess, a position of the chess board which is not reachable from the starting position of the game. Herman Weyl [1927] has compared such non reachable positions with independent propositions in a mathematical system. Motivated by this similarity can we say that mathematics is a game? Someone is trying to prove a theorem in arithmetic. Maybe it

is neither possible to prove this theorem, nor to find a counter-example of it, i.e. to disprove it by showing that its negation is a consequence of the axioms of arithmetic. The theorem is maybe independent, but it is possible to prove the independence of a theorem, such as an independence of the axiom of choice relatively to the axioms of ZF. An independence proof can be seen as end of the game. En passant let us note that Tsuji, da Costa and Doria [1998] have proven some interesting results of independence in theory of games by However a reason, among others, to say that mathematics is not a game would be to say that we can use neither chance nor a mechanism to win the game, i.e. to prove something, since chance is not considered as a mean of proof and there is not a general algorithm that can be apply to find a $proof^4$.

But can say that something which is purely algorithmic/mechanical is a game? After the birth of deep blue, can we still say that chess is a game? We can say that what is interesting is not to play algorithms but to find algorithms, and this can be considered as a game or as part of the game. This is connected with strategic rules.

Strategic rules

A strategy is a plan to develop an action. For example, in the famous movie *Rules of the game* by Jean Renoir, the following classical strategy of seduction is expounded: "Quand une femme rigole, elle est désarmée, vous en faites ce que vous voulez" (when a woman laugh, she is disarmed, you can do what you want of her).



⁴However theorems can be proved by algorithms. This was the case of a favorite conjecture of Tarski about Boolean algebra. Boolean algebra is an undecidable theory, but an algorithm to find a proof of this conjecture was run by a computer and after some time, it gave a positive solution with a proof. This is considered as the first true computer proof performed by a computer alone (see [Cipra, 1999]).

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In football, the coach can consider that four players are attacking, in chess one may perform a Réti opening, in monopoly one may decide to buy everything he can buy. These ways of playing are not rules specified in the official regulations of the games. They correspond to optional actions, we can call them rules because they are regulating the game in the sense that the player will conduct his behavior according to them. Strategic rules depend not only on deontic rules, but also they depend on teleological rules: they are applied to win. Even in a pure gambling game there are rules of strategy: someone may always play his date of birth, luckily enough he may win before his 100th birthday.

To play a game is not only to apply allowance rules or developing hazardly or skillfully actions which are not prohibited, to play is also to create rules. That is maybe the most interesting aspect of a game and it is related with other human and non-human activities. François Le Lionnais and Raymond Queneau, the founders of the Oulipo, have argued that what is interesting in literature is not to write according to existing rules (such as alexandrine in poetry, the climax in drama - happening at the minute 69 in any good Hollywood movie) or reject rules (Surrealism), but to create new rules and write according to them, more generally: the seeking of new structures and patterns which may be used by writers in any way they enjoy [Oulipo, 1973]. One of the most famous Oulipian product is the book by Georges Pérec (1978), *La vie mode d'emploi (Life a user's manual*).

Most of the oulipians are mathematicians (such as Claude Berge) or have a mathematical background (like Le Lionnais en Queneau themselves), this does not mean that they are just applying mathematical rules to literature, but rather that they are inspired in the game of literature by mathematics where creation of rules is also a fundamental feature (about philosophy of mathematics, we recommend the masterpiece edited by Le Lionnais, *Les grands courants de la pensée mathématique* [1948], with papers by Bourbaki, Le Corbusier, etc). We can see for example Cantor's diagonal argument as a new rule. In this proof Cantor is using some rules of reasoning such as the excluded middle, but the form of the proof itself is new and this new idea can be generalized and applied to develop new ways of reasoning, in this sense it is a rule, or a new way to use the usual rules. As pointed out recently by Brady and Rush [2008], if we change the rules of the logic game, then we cannot perform anymore Cantors argument.

Someone may argue that strategy is at the meta-level: it is how to use rules. And for this reason, one may be radical saying that strategic rules are not rules but meta-rules. Here we consider that strategic rules can be either rules about rules or rules tout court. Even in logic the distinction between rules and metarules is not absolute, from a certain point view the deduction theorem can be seen as a meta-rule, from another perspective, it can be seen as a rule (see [Beziau, 1999] about this discussion). For us a meta-rule has some common features with a rule and in particular we agree with the possibility of formally describing strategic rules, as developed by Batens and Provijn [2001, 2009].

2.3 Rules and the four Aristotelian causes

It is possible that nature is acting according to strategic rules, and that new rules are being created all the time. The main distinction between nature (the

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universe) and a game is that the notion of teleological rules is not clear, unless we consider a God with a secret objective or the universe as a frescobol game: the goal being to stay in the air ...

To finish we present a connection between Aristotle's four causes and the four kinds of rules we have described. We will not here enter in details or justifications but leave it as a source of inspiration for future work. Here is a table:

Framework rules	Material cause
Deontic rules	Formal cause
Strategic rules	Efficient cause
Teleological rules	Final cause

TABLE 2 : THE FOUR KINDS OF RULE AND THE FOUR ARISTOTELIAN CAUSES

The value of this table can be checked by using it on the one hand to apply Aristotle's theory to some specific games and on the other hand by establishing a connection between rules of the games and activities other than games.

If we consider that the Aristotelian theory of four causes explains everything, then we may argue that everything can be explained by these four categories of rules via this concordance table. But can we then jump to the claim that a tsunami, the solar system or whatever going around in the world is a game?

Not necessarily, because a game is not only characterized by rules, we have also characterized games with a chemical formula. And it is not obvious that such phenomena obey this chemical formula.

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