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What is Semantics?
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Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we say is true.

Bertrand Russell

INTRODUCTION

In what follows, we present, in a rather rough and preliminary way, some general remarks on a quite delicate issue: semantics. To some extent, as will be clear anyway as we proceed, we are here concerned with formulating and spelling out some questions, problems and ideas on this topic, rather than considering their possible solutions. Our basic aim thus consists in just pointing out to some problems that, as far as we see, deserve to be considered and examined — a project, in fact, for a series of works. This explains, or so we hope, the rather concise style adopted throughout the piece.

After some general considerations, made in section 1, we shall briefly present, in section 2, nine thesis on semantics.

Before continuing, however, we wish to add a last introductory remark. It consists in stressing the considerable departure found today between the original sense of this term («semantics») and its current, rather multiple uses. This fact, however, by no means reduces itself to a matter of words. Underlying this meaning variance, it is possible to identify, as far as we can evaluate, a strange shift on the main direction of the semantic analysis of a formal system. As a result, it seems to us, some very important conceptual questions are not correctly spelled out — or not even perceived. Our main purpose now is to call the attention upon them.

1. LOGIC, SEMANTICS, SET THEORY

When first proposed in the fields of logic and formal sciences, the term «semantics» used to present a clear sense. It was supposed to denote that part of an analysis of a language concerned with the determination of the meanings of its (well formed) expressions. (On this regard, see the interesting comments as well as the references presented by Church, in a section dedicated to semantics, at the end of his introduction to the celebrated Church [1946].) More recently, however, faced with an enormous variety of alternative meanings, it is no longer possible to specify an exact sense to this word. Indeed, the process of stretching its meaning has reached such a point that even a *semantic* conception of theories, within the philosophy of science, has recently been advanced!

In intimate connection to this point, our first remark stresses the fact that, as far as the earlier sense of semantics is concerned, Tarski's set theoretical semantics is *not*, in a strict sense, a semantics: it just represents an extensional association between, on the one hand, terms and predicates of a language to, respectively, particular objects and classes of objects of a fixed domain, on the other (this point, indeed, was already noticed by Church himself). By no means the meaning of these terms and predicates is established this way: no intensional factors are taken into account!

More importantly, however, on this regard, is perhaps to note that a set theoretical semantics for a non-classical logic (e.g., relevant or paraconsistent logics) — besides not being, strictly speaking, a semantics —, being constructed within classical set theory, it reveals itself, from a philosophical perspective, completely unsatisfactory. One reintroduces, so to speak, by the backdoors, exactly what was intended to be left on the entrance!

That is the reason why one of the authors (Newton da Costa), when first developed his paraconsistent systems, presented them through a syntactical approach. At that time (1954), not having yet a paraconsistent set theory at his disposal, it would not be possible to articulate a reasonable (set theoretical) semantics for that logic.

(It should be noted, and we shall return to this point later, that in order to have a logic minimally developed, at least *three* conditions must be met: besides the formulation of a propositional calculus, a quantificational theory is to be advanced; furthermore, the same shall be stated for a set theory. Thus, Smiley and some other reputed forerunners of paraconsistent logic, despite the undeniable relevance of their work, have not elaborated, strictly speaking, such a logic: more should had been done.)

More generally, the usual set theoretical semantics, given the way it is articulated at present, depends on its underlying set theory: if one changes such a theory, the semantics itself is, *ipso facto*, changed. In particular, the same is the case for Tarski's definition of truth.

A last word. As some recent researches within valuation theory has shown (see Grana [1990]), every logic admits a two valued semantics. The question then naturally results: in what sense can we talk of a sound *semantics*?

2. SOME THESES ON SEMANTICS

Based on some of the previous remarks (though not only on them), we shall now concisely present nine distinct, but interconnected, thesis on semantics.

1. There is no radical semantics — in the sense of a presuppositionless one. As we have already remarked, Tarski's semantics, for instance, depends on the particular set theoretical setting within which it is formulated. However, there seems to be a kind of «intuitive semantics» underlying our standard semantical constructions. Nevertheless, it is employed just on heuristic, and by no means justificationist, grounds. Its role consists in supplying some theoretical guidelines in order to help us in obtaining our semantical results.

2. As far as we understand, and returning to an earlier point, in order to have strictly speaking a *logic*, one has to present definitions of the notions of *demonstration* and *thesis* adequate to (1) a propositional calculus, (2) a quantificational theory (with identity), and eventually (3) to a set theory. This indeed is an important constraint, given that one of the roles of a logic consists in supplying some tools in order to assist us in the development of conceptual systems. Frequently, however, in various domains, such a development depends on the adoption of particular set theories — and here comes the need of them. It is plain that such a remark is undeniably straightforward as far as scientific contexts, both in formal as well as in empirical domains, are concerned. If one intends to develop mathematics, physics or some further scientific field, set theory, in some or other form, as it is obvious, is probably to be employed. Thus, if your most beloved logic is to be of any use within this process (and it seems fairly reasonable to suppose so, or at least to intend that), then the best you can do is to have it developed up to a set theoretical level.

Given these remarks, we may conclude that, strictly speaking, there might not be a relevant logic. Indeed, at least as far as our current knowldge is concerned, it is not possible to develop a strictly *relevant* set theory. (As is known, it is not even possible to demonstrate, based on this logic, the unicity of the empty set, for such a proof depends on the fact that $A, \neg A \vdash B$.) Furthermore, in connection to this point, and granting that relevant logic is a logic, given the non existence of a relevant set theory, we wonder if, from a philosophical perspective, it is legitimate simply to adopt, as it is usually done, in order to formulate a semantics for this logic, classical set theory, which, as we know, is constructed based on classical, and not on relevant, logic. Unfortunately, this move seems to be rather puzzling, given the relevant theorist's rejection of classical logic.

3. A convenient logical system, as far as contemporary science is concerned, should somehow contain classical logic and its semantics — or, at least a considerable portion of it. Otherwise, on the one hand, some basic scientific applications would not be developed, nor, on the other, some aspects involved in the construction of mathematics will be possible. This point, indeed, was already noticed by Hilbert himself. In fact, though perhaps being a bit hasty in his generalization regarding the role of Aristotelian laws of logic in the construction of mathematics, from his viewpoint:

> [...] we cannot relinquish the use either of the principle of excluded middle or of any other law of Aristotelian logic expressed in our axioms, since the construction of analysis is impossible without them. (Hilbert [1927], p. 471).

4. Classical logic, just as relevant logic, is based on a certain kind of semantic atomism: under certain contexts, a particular proposition is true or false independently of any other. (Wittgenstein appears to have adopted such an assumption in his *Tractatus.*) Physics, in this sense, seems to be committed to this kind of atomism. This, anyway, would be a limitation to classical logic.

On this regard, how to apply relevant logic to dialectics, if the latter assimilates everything to a «fluid»? And to the coherence theory of truth? In these cases, are classical semantical construction sufficient?

5. We can imagine a logic in which every proposition depends, for the determination of its truth-value, on the propositions in its neighborhood. So, we would have a «neighborhood» semantics, quite different from the classical one.

6. Given that classical logic is contained within some paraconsistent logic, the former can be employed in order to supply a semantics for the latter. Furthermore, it is possible to construct *paraconsistent* set theories. Thus the argument just presented agains relevant semantics can not be directed to paraconsistent logic.

7. Paraphrasing Einstein's celebrated remark, we can say that the propositions of logic, as far as they are true, do not apply to reality, and as far as they apply to reality, they are not true. This is the case when logic is not conceived in an absolutist way, but as a theory among further theories. There are at least four arguments for such a claim: (1) the plurality of logics; (2) the apparent fuzziness of reality; (3) the fact that formal sciences (in particular, logic) are human constructions; and (4) the opposition between logical rigour and the fuzziness of reality. A good semantics has to cope with all of these issues.

8. How to choose between alternative logical and semantical systems? To put it in a nutshell: through an examination of its consequences. Obviously, some pragmatic aspects are also to be considered here. Being more specific, to some extent, we make our choices based on various considerations that can however be divided into two classes: (1) *formal* requirements, and (2) *material* conditions. Regarding (1), we find the usual formal constraints on a logical system: its soundness and completeness, its relative consistency and so on. (The failing of some of these conditions may, to some extent, present some negative evidence against the system.)

Concerning the material conditions, one may present some criteria of choice roughly based on the following grounds: (2.1) *heuristic aspects* of the system (as far as its deductive power is concerned, for instance), (2.2) its *problem-solving resources*, (2.3) its *adequacy* in order to make sense of some scientific applications and some scientific strategies of reasoning.

9. Someone may present the following question: what is the usefulness of paraconsistent logic and its semantics? To such a question we may reply with a further question: what is the usefulness of classical logic, if we have paraconsistent logic which (at least in some systems) contains classical logic?

3. CONCLUDING REMARK

As it might be easily noted, it seems to us that if the preceding theses are nearly correct, Russell's comment on mathematics, presented above, with obvious changes can also be rather naturally applied, in particular, to *pure semantics* itself.

REFERENCES

Church, A. [1946]: *Mathematical Logic.* Princeton: Princeton University Press.

Grana, N. [1990]: *Sulla Teoria delle Valutazioni di N. C. A. da Costa.* Napoli: Liguori Editore.

Hilbert, D. [1927]: «The Foundations of Mathematics», English translation of the original 1927 German paper reprinted in van Heijenoort (ed.) [1967], pp. 464-479.

van Heijenoort, J. (ed.) [1967]: *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931.* Cambridge, Mass.: Harvard University Press.

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