An Analogical Hexagon

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Introduction

Analogy is a very famous and popular notion. Everybody likes to make some analogies. Roughly speaking making an analogy is to compare two *different* things, stressing one *similar* feature, which is *transposed* from one thing to another one, shedding a new light on it. Considering this transportation, we can consider that analogies are metaphors (cf. the etymology of "metaphor"). Making analogies is an art, the result can be a *chef d'oeuvre* or an ugly and ridiculous thing when the mayonnaise is not succeeded.

Analogies are most of the time a bit challenging if not controversial, even when performed by a good writer like Leon Tolstoi: "Truth, like gold, is to be obtained not by its growth, but by washing away from it all that is not gold", or a virtuosic mathematician like Stefan Banach: "A mathematician is a person who can find analogies between theorems; a better mathematician is one who can see analogies between proofs and the best mathematician can notice analogies between theories. One can imagine that the ultimate mathematician is one who can see analogies between analogies".[1]

Analogy looks in some way as opposed to rationality. But an analogy is not necessarily completely irrational. "Ana-logon" is not "a-logon". "Ana" means above. Although Pato's cave can be seen as an analogy, there is in ancient Greece a much more precise view of analogy, this is *proprotional analogy*: A is to B as C is to D. Prade and Richard have recently provided a detailed logical theory of it (see [28]). For that they used the theory of opposition, in particular the square and the hexagon of opposition.

In the present paper we are also using the theory of opposition, but consider that though analogical proportion is part of analogy. it does not reduce to it. We present a hexagon of opposition figuring analogy in a wider sense. This hexagon is not constructed by breaking the dichotomy identity/difference into a trichotomy but by inserting it in a framework with two more dichotomies and a trichotomy involving opposition itself.

In a first part we discuss the different ways to go beyond dichotomies, using trichotomies and hexagons of opposition. We explain the theory and give some examples. In a second part we show how to produce a hexagon with analogy. In a third we investigate the meaning given to analogy with this hexagon, discussing the related notions which appear in the hexagon, in particular similarity, and presenting some examples.

In this paper, as in our recent paper "Possibility, imagination and conception" [9], our methodology has three aspects: structurality, equilibrium between descriptivity/normativity, prototypical examples (we have discussed this three-fold methodology in the introduction of the mentioned paper, so we will not discuss it again here) and we are also making extensive use of images. This paper is self-contained and can be considered as an introduction to the theory of opposition.

[1] This is a rather metamathematical analogy, the Bourbachic mathematician André Weil wrote a letter to his sister about a true mathematical analogy, see [29].

1. From dichotomies to hexagons

1.1. Why breaking the dichotomy?

Pythagoras' table of opposites was an interesting departure point for the systematization of thought:

One	Many
Right	Left
Male	Female
Square	Oblong
Light	Darkness
Good	Bad
Straight	Crooked
Rest	Motion
Close	Open
Happy	Sad
Odd	Even

Fig. 1 - PYTHAGORAS' TABLE OF OPPOSITES

As we have pointed out in previous works, according to this table a pair of opposites is not the by-product of a negational artificial construct, whether extensional (complementation acting on sets of objects, extensions of concepts or properties) or intensional (negation operator acting on propositions, intensions of concepts or properties). It is possible that historically in fact the things happened the other way round: from a set of dichotomic pairs emerged the notion of classical negation. The two sides of the pair have a positive meaning, this is a feature we have to keep in mind if we want to break the dichotomy in a trichotomy, which will then not be just an abstract nonsense.

Once the notion of abstract dichotomy has been put forward, it is easy to generalize it into a trichotomy, a quadritomy, a pentagony, ... any kind of politomy. We can generalize the bi-partion of the universe or a cake into any n-partition: dividing again and again, *ad infinitum* ...

Why breaking the dichotomy? In many cases a dichotomy sounds artificial, just to take one classical case: Republicans vs Democrats in the United States Political system. Are the two sides really different? Why are there no other options?

Fig. 2 - POLITICAL DICHOTOMY - MADE IN USA

On the other end polytomy may lead to chaos. In Brazil there are more than 30 political parties and the differences and similarities between any two of them is difficult do specify.

Fig. 3 - POLITICAL POLYTOMY - MADE IN BRAZIL

1.2. Contrary trichotomy

Between dichotomy and cacotomy it is important to find a nice equilibrium. Three is a good number for physical equilibrium: a chair or a table with two legs does not stand up, with four legs it can be wobbly, with three legs it is stable. In many cases three seems enough, from the point of view of thought and/or reality and also action.

The world has been famously divided in three by Alfred Sauvy (*L'Observateur* August 14, 1952):

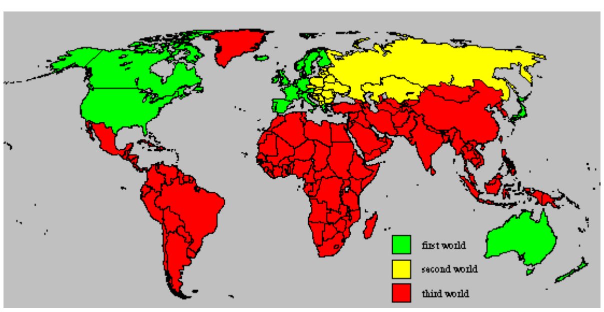

Fig. 4 - TRI-PARTITION OF THE WORLD

They are three primary colours: Red, Blue, Green. These colors appear on the flag of the Republic of Karelia (There are many flags that have three colors).

Fig. 5 - FLAG OF THE REP. OF KARELIA / TRI-PARTITION OF COLORS

A tri-partition can also be presented in a form of a round pie. This is nice to compare proportions. Below a picture comparing the evolution of the three classes of ages of American population.

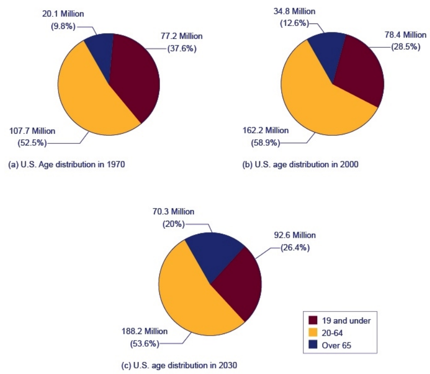

Fig. 6 - EVOLUTION OF THE THREE CLASSES OF AGES IN THE USA

It is also possible to use triangles. Space and time can both be conceptualized by triangles:

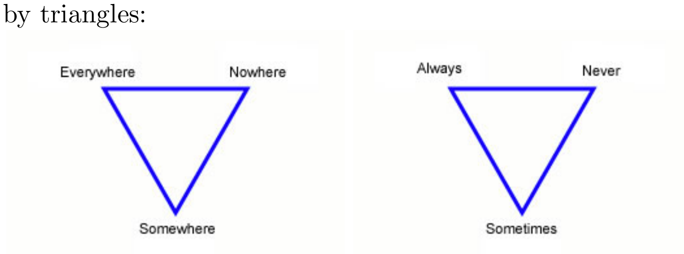

Fig. 7 - TRIANGULATION OF SPACE AND TIME

And below two triangles corresponding to two tri-partitions of actions:

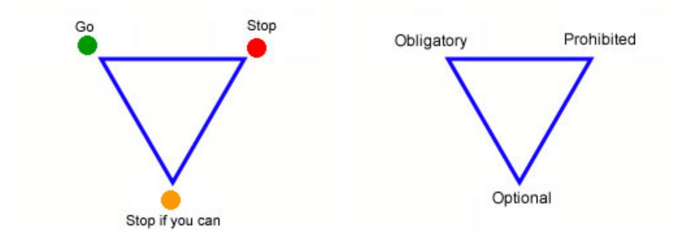

Fig. 8 - TRIANGULATION OF ACTION

Trichotomies can be reprensented by different diagrams. A plan figure (rectangle, circle, whatever) divided in three regions or a skeletal decorated triangle. The first option is good because it explicitly shows that we are dealing with tri-partition: the three regions are not overlapping and they are covering the whole surface. It also allows to represent the different proportions. It pushes on extensionnality. Triangulation is more abstract but it allows us to go further on and also on a more intensional direction if needed.

The trichotomies we have presented here are trichotomies of *contrariety*. Extensionally the idea of contariety can be defined using any partition which is not a bi-partition. Any two regions (members) of such n-partion ($n > 2$) are said to be contrary to each other, by contrast with the two sides of a bi-partition which are said to be *contradictory*.

Considering propositions (in particular embedding some properties or concepts, e.g. "The car is red", "It is always raining in London", "It is obligatory to vote"), we say that two propositions are contradictory iff they cannot be true together and cannot be false together and that two propositions are contrary iff they cannot be true together but can be false together.

The notion of contrariety was already put forward by Aristotle, breaking Pythagoras' dichotomy, but up to now the notion is not very popular in the sense that there is not a word in ordinary language for it and that it is generally confused with contradiction. For example many people will give as a typical example of contradiction, a round square, but circles and squares do not form a bi-parttion of the universe of geometrical figures, for example a triangle is neither a triangle, nor a square –See our recent papers "Round squares are no contradictions" (2015) and "Disentangling contradiction from contrariety via incompatibility" (2016). In this second one we propose to systematically use the word "incompatible" for the conjunction of contradiction and contrariety. This is a disjunct conjunction because two things cannot be at the same time contradictory and contrary. If we say that two things are incompatible, this means that they are either contradictory or (exclusive or) contrary without further specification.

The notion of triangle of contrariety has been emphasized by Robert Blanché (1898–1975). His seminal book is *Structures intellectuelles - Essai sur l'organisation systématique des concepts*, published in 1966 [22]. His first papers on the theory of opposition are *Quantity, modality, and other kindred systems of categories*, published in *Mind* in 1952 [19] and *Sur l'opposition des concepts*, published in the Swedish journal *Theoria* in 1953 [20].

Blanché did not stop with triangles, he went further on, not with squares or pentagons, but with hexagons. But not because he wanted to multiply the numbers of sizes, to have some polygons of contrariety corresponding to n-partition ($n > 2$) of contrariety. Blanché's hexagon is constructed by dualization as a product of two triangles. It is a prolongation of triangulation.

1.3. The star and the hexagon

From a blue triangle of contrariety, considering the contradictory of each of its corners, we can build the following star:

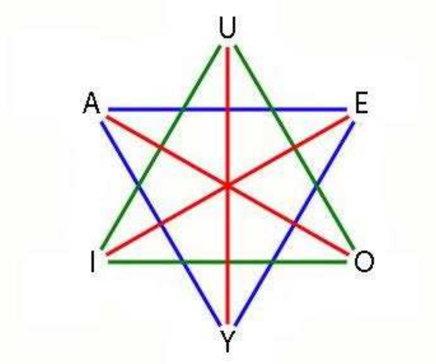

Fig. 9 - STAR OF OPPOSITION

The blue lines represent the notion of contrariety and the red ones the notion of contradiction. What about the green ones? This is the notion of *subcontrariety*. If we say that "It is not prohibited to vote in the Republic of Karelia", it means it is either obligatory or optional, in other words, it is allowed. Two subcontrary propositions can be true together but cannot be false together. A triangle of subcontrariety does not correspond to a tri-partition, because the notions are exhaustive, they are in fact exhaustive two by two, but they are overlapping (this does not correspond to the set-theoretical technical sense of partition). Two subcontrary concepts are exhaustive but not exclusive. For example irrational numbers and algebraic numbers are subcontrary in the reals: every real number is either algebraic or irrational or both (like the square root of 2). Each opposed corner of the triangle of contrariety is the union / disjunction of two other corners. By the very nature of the triangle of contariety it is a disjunct union / exclusive disjunction. We can complete the star of opposition by adding some black arrows describing explicitly which pairs correspond to which corners:

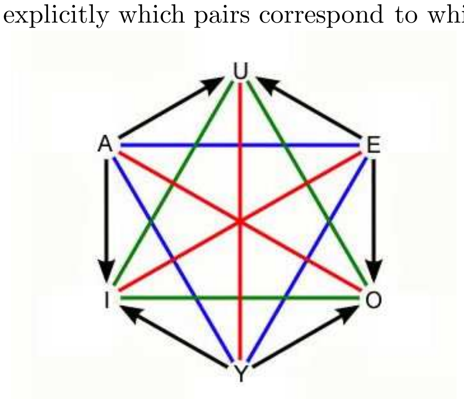

Fig. 10 - HEXAGON OF OPPOSITION

The black arrows correspond to inclusion / implication. The I corner is the disjunct union of the A and Y corners. The U corner is the disjunct union of the A and E corners. The O corner is the disjunct union of the E and Y corners. Here is a particular instantiation of the hexagon:

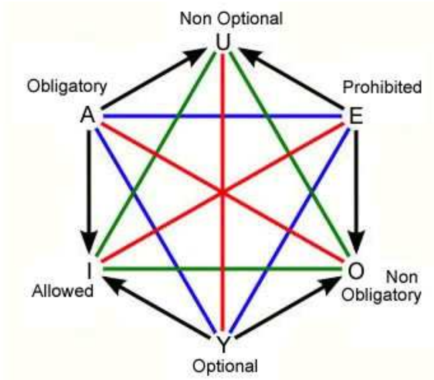

Fig. 11 - THE DEONTIC HEXAGON

The letters "U" and "Y" were chosen by Blanché who built such a hexagon; "A", "E", "I", "O" are part of the tradition related to the famous *square of opposition* popping up at the middle of the hexagon.

1.4. Recovering the square of opposition and the meta-hexagon of opposition

Blanché's hexagon can be seen as a reconstruction of the square of opposition, or better, a recovering of it, since there were several problems with this square, solved by Blanché's hexagon. In particular problems with quantification. We have discussed this in details in our paper "The power of the hexagon" (2012) [5]. Initially the square was about quantification. More specifically about the theory of quantification from the viewpoint of Aristotle's theory of categorical propositions. The square was not presented by Aristotle himself (although he suggested it in some sense, see [25]). Apuleius and Boethius developed it explicitly, probably from a common prior source (see [23]). Here is Blanché's hexagon of quantification:

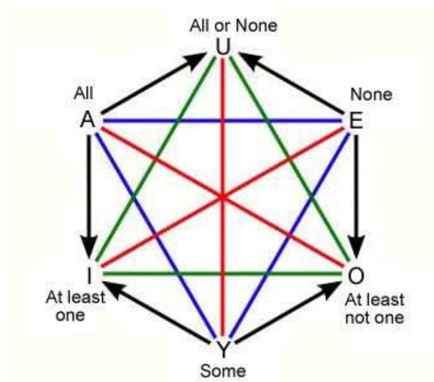

Fig. 12 - THE HEXAGON OF QUANTIFICATION

Note that within a hexagon there are not only one square of opposition but three squares of opposition It is possible to see that by rotating the hexagon. In the above hexagon, besides the square:
< All – None – At least one – A least not one >
there are the following two other squares:
< Some – All – All or None – At least not one >
< None – Some – At least one – All or none >.

A square of opposition has the following structure:

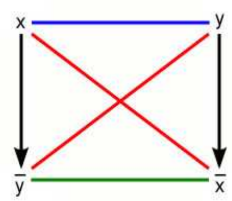

Fig. 13 - ABSTRACT SQUARE OF OPPOSITION

We have given the names x, y, $\overline{x}$, $\overline{y}$ to turn the names of the corners more anonymous, more variable, and at the same time more specific. We could have chosen the following configuration:

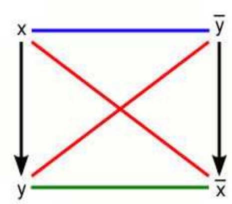

Fig. 14 - ABSTRACT SQUARE OF OPPOSITION - Beta version

But our choice was to emphasize the triangle of contrariety x, y, z which is inside the following hexagon:

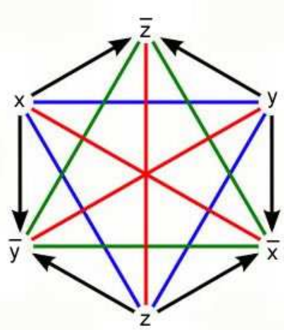

Fig. 15 - ABSTRACT HEXAGON OF OPPOSITION

The theory of the square of opposition is based on three notions of opposition that we have represented in blue (contrariety), green (subcontariety) and red (contradiction).[2] These three notions of oppositions themselves form a triangle of contrariety:

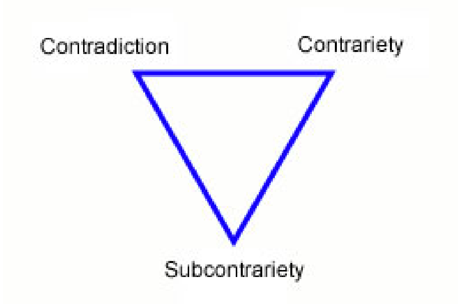

Fig. 16 - META-TRIANGLE OF CONTRARIETY

We have recently (see [10]) proposed the following linguistic decoration of this meta-hexagon, naturally generated from this meta-triangle, including a call for a missing name for one corner:

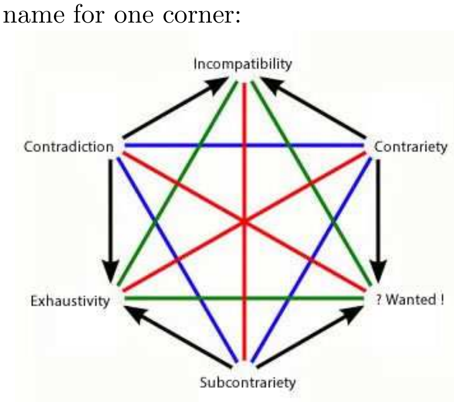

Fig. 17 - THE META-HEXAGON OF OPPOSITION

1.5. Two ways to construct a hexagon breaking a dichotomy

We can break a given dichotomy by splitting it into a trichotomy finding a third term. This is quite easy, especially if we do this on basis of quantity or degree. For example there is the rich, the poor and the middle citizen. We can also consider the very rich, the very poor, and so on. In Brazil the population is indeed standardly divided into 5 classes, A, B, C, D, E. We can do this with most everything, for example temperature: hot, cold, very hot (burning), very cold (freezing), tepid.

[2]We have introduced these colors for representation of the diagram in our paper "New light on the square of oppositions and its nameless corner" (2003) [2] which was our first paper explicitly on the subject.

What is more interesting is to find three different qualitative states. In the economical realm, we can break the dichotomy sell–buy into the trichotomy sell–buy–rent. Renting is neither buying (unless it is leasing) nor selling.

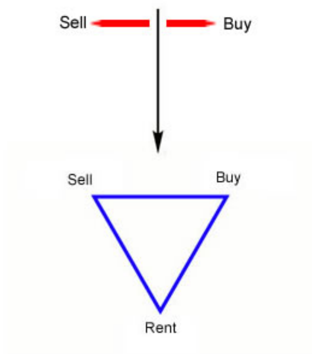

Fig. 18 - SPLITTING THE DICHOTOMY BUY / SELL

From this trichotomy it is not easy to construct a meaningful hexagon, i.e. to find positive characterizations of the three corners of the dual subcontrary hexagon. What is for example the notion and name corresponding to buying or (exclusive or) selling ?

The situation is easier when splitting the dichotomy music–noise into the trichotomy music–noise–silence

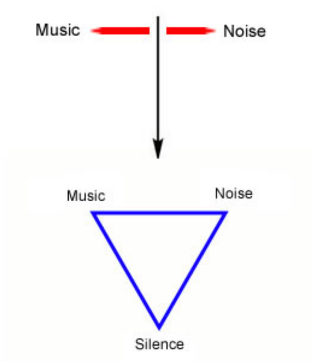

Fig. 19 - SPLITTING THE DICHOTOMY MUSIC / NOISE

because it makes sense to qualify the (disjoint) union of noise and music as *sound* and we can tentatively qualify the (disjoint) union of music and silence as *harmony*. We are left with the union of noise or silence that is not too easy to positively qualify and that we can name just "non music".

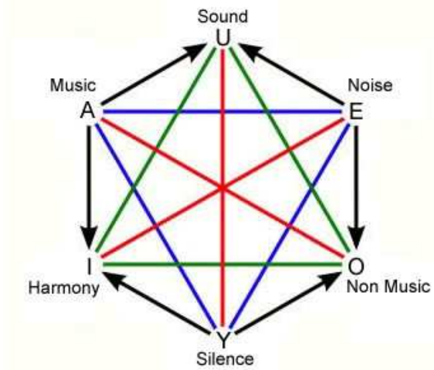

Fig. 20 - THE MUSICAL HEXAGON

Transforming a dichotomy into a trichotomy does not always work, a dichotomy can resist, it can be a true dichotomy. But nevertheless not necessarily an absolute dichotomy. By merging a dichotomy into a more complex network, we don't split the dichotomy but we relativize it. Let us consider one example.

In semiotics we have an opposition between arbitrary signs and non-arbitrary signs. Arbitrariness is qualifying here the relation between the sign and what it is pointing at, its signification. An expression constructed with a latin alphabet like "turn right" is an arbitrary (group of) sign(s). By contrast the following traffic sign is not arbitrar:.

Fig. 21 - A NON ARBITRARY SIGN

Of course there are degrees of arbitrariness and the name for non-arbitrary signs is not completely determined. It makes sense to call them "symbols", in particular considering the etymology of this word (see [16]). But Peirce made a distincion between "symbols" and "icons", considering that in the case of a symbol, the connection between the sign and its signification is conventional. Since a convention is in some sense arbitrary maybe it is not clear if a symbol according to Peirce has to be put on the side of arbitrary or non-arbirary signs. To create a third category would be a bit absurd. We prefer to keep using the word in its original etymological sense and turn the dichotomy arbitary–non arbitrary signs in a more meaningful dichotomy arbitrary signs–symbols. But we can consider that there are variations among symbols and also among arbitrary signs. We may then preserve this dichotomy by introducing a specific case of symbols, those literally picturing the signification, like a photo; we call them icons (Our position is not diametrically opposed to the one of Peirce, it can be seen as a variation of it). We have then the following situation:

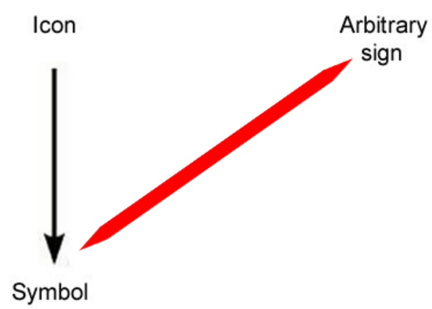

Fig. 22 - ACTING ON THE DICHOTOMY ARBITRARY SIGN/SYMBOL

from which we draw the hexagon:

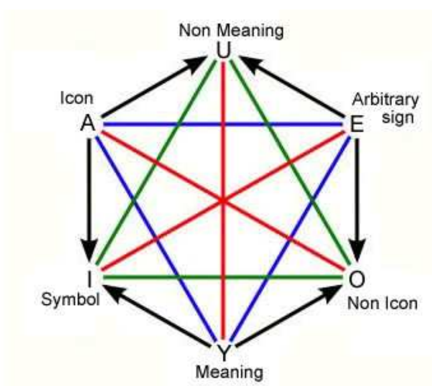

Fig. 23 - THE SEMIOTICAL HEXAGON

We have not split the dichotomy arbitrary signs-symbols but we have have put it in a more general framework with two other dichotomies: meaning–meaningless signs, iconic–non iconic signs. And these three dichotomies are precisely related with each other within a hexagonal structure.

2. Delivery of an analogical hexagon

2.1. Squaring identity and difference - First attempt

Identity vs. difference is one of the most famous dichotomies, however not part of Pythagoras' table of opposites. Maybe because it is the most general and the most abstract one. More fundamental than *one and many*: multiplicity presupposes difference, it is a particular case of difference.

We can gradually break this dichotomy, considering things which are more or less identical, more or less different. By doing that, like with other graduated breaks, we are back in fact with a dichotomy: on the one hand a cloud of identical things whose extreme is pure identity, on the other hand a cloud of different things whose pure extreme is pure difference, what is fuzzy is the border between the two.

It seems that there is no real qualitative way to turn identity and difference compatible. What about something which is neither difference, nor identity? Someone may argue that some things are incomparable, that for example 4 and 7 are different but that their difference is not the same difference as the one between 4 and *Donald Duck*. Both of these entities are maybe fictional, but *Donald Duck* is a duck and 4 is a number. We can say that *Donald Duck* and 4 are *incomparable*.

Fig. 24 - INCOMPARABLE THINGS

We can indeed make such a distinction but considering that these are two kinds of difference. Difference is a fairly general notion and there is no reason to ontologically restrict it, especially if this restriction is based on difference itself: incomparability is a difference between things of *different* nature.

From the subalternation incomparability → difference:

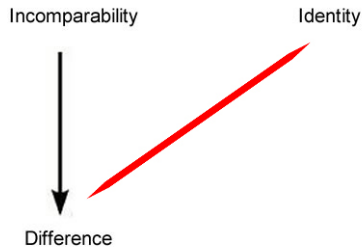

Fig. 25
FIRST ACTION ON THE DICHOTOMY IDENTITY / DIFFERENCE

we have the following square of opposition:

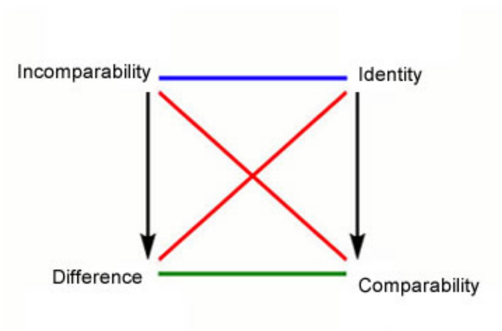

Fig. 26 - FIRST SQUARING OF IDENTITY AND DIFFERENCE

But having analogy in view, this is not very satisfactory, because the Y corner of the corresponding hexagon, which would be the closest location for analogy, is still far from it: it is different and comparable. Comparability is nice but too far away from identity.

2.2. Squaring identity and difference - Second attempt and the resulting hexagon of analogy

There is a more interesting difference among differences, this is opposition itself. If two things are different, they are not necessarily opposed. This distinction in fact is very important to understand what opposition is. On the basis of it we can in particular clearly exclude subalternation from the family of oppositional relations. Subalternation corresponds to (strict) inclusion / implication. Cats are part of the feline species and it makes no sense to say that cats and felines are opposed. On the other hand we can say that these two classes are different, since not every feline is a cat. If something is obligatory, it is allowed. Allowance and obligation are not opposed. Obligations are a proper kind of allowances. Subalternation is a fundamental tool in the theory of opposition. It was originally present in the square and it is also very useful in the hexagon, but it is not a member of the 3-part oppositional world. This is why when putting colors, we decided to paint it in black.

With the subalternation opposition → difference and the dichotomy difference–identity

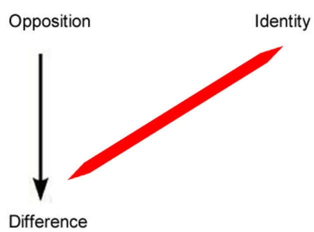

Fig. 27
SECOND ACTION ON THE DICHOTOMY IDENTITY / DIFFERENCE

we generate the following hexagon:

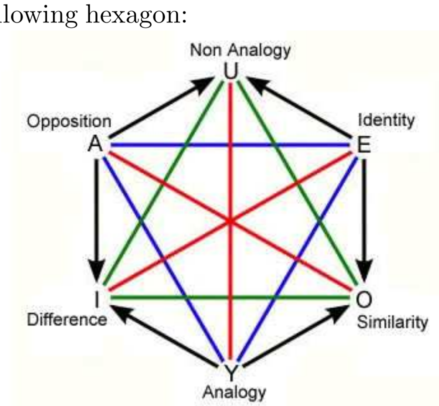

Fig. 28 - THE ANALOGICAL HEXAGON

This hexagon is nice because we have analogy as the product of difference and similarity On the other hand, at first sight, it is not so nice because of the contradictory opposition between similarity and opposition. Many times opposed things are very similar. But there is a way to sort out this difficulty: it is to consider that this kind of similar oppositions are in fact pseudo or illusory oppositions. Moreover we can firmly defend the position of similarity in the South-Oriental position of the hexagon, considering that it is a subaltern of identity.

2.3. Triangle of analogy

When constructing a hexagon, not by intertwining two triangles, but from a square, it is interesting to closely analyze the triangles inside the hexagon, in particular the blue triangle of contrariety. Here we have:

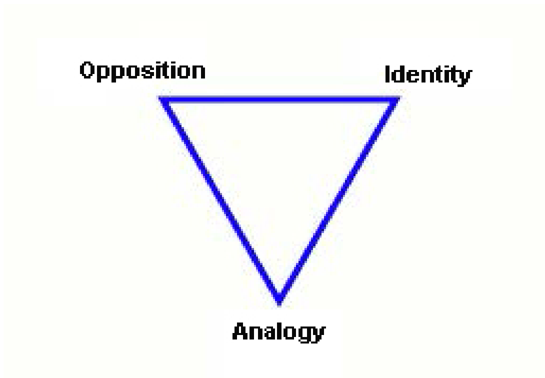

Fig. 29 - ANALOGICAL TRIANGLE

Let us examine if this triangle has the three central features of the theory of triangulation: quality, incompatibility, exhaustion.

- Quality: has each vertex of this triangle its proper quality? Is none of them a variation of degree of another one?
- Incompatibility: are each of the three pairs of vertices incompatible?
- Exhaustion: is there any notion outside of this triangle?

In some sense, and this is how it appears in this triangle, analogy is in between opposition and identity, but since analogy is a mix of the negation of the two it would be difficult to argue that we are gradually going from identity to analogy, or from opposition to analogy. There is at some point a double jump, at discretion. Turning the triangle we can repeat the same argumentation.

Incompatibility between opposition and identity can be characterized by saying that identity is a reflexive relation and opposition is anti-reflexive, this clearly appears if we consider the three notions of opposition of the theory of opposition: a proposition is neither contradictory, contrary or subcontrary to itself. We can use the same argument for the incompatibility between opposition and analogy if we consider that analogy is reflexive. Thinking of properties (or axioms) for these relations, one may want to argue that identity and analogy are incompatible on the basis that transitivity holds for identity but not for analogy. But this is a wrong argument, because analogy is not anti-transitive. We can simply consider the fact that if two things are analogous, they are different and since difference is incompatible with identity, so analogy is incompatible with identity.

Considering exhaustion, someone may say that difference is missing. But the reply is pretty simple, we can just say that two things are different either if they are analogous or opposed, difference is split in two. The only serious challenge seems incomparability. Can we say that *Donald Duck* is: opposed to the number 4? identical to the number 4? analogous to the number 4? None of the choice seems satisfactory. One possibility would be to transform our trichotomy into a quadritomy. There is a way to do it in fact preserving ternarity, using the tetrahedron below which puts together four triangles of contrariety and can be itself considered as a kind of three-dimensional triangle (any quatritomy of contrarieties can be dribbled in this way - this technique has been developed by the famous Italian footballer of opposition Alessio Moretti).

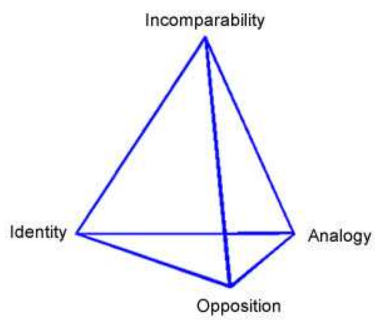

Fig. 30 - ANALOGICAL TETRAHEDRON

If we want to stay on a flat land and keep ternarity, we can simply discredit incomparibility. How to do that? We can say that it is not relevant. In general when we are building a trichotomy, a hexagon, we don't want to put the whole world in it. In our hexagon of music, we have not included ducks. We want to work with things which are at the same level, in the same field. Here of course the situation is more tricky. But, funny enough, we can use comparability to exclude comparability.

3. Prototypical examples

There are different ways to characterize a notion, for example by comparing it to other notions, this is obviously part of our methodology here. Another way is to represent a notion by a prototypical example. This is not necessarily easy. It is in fact particularly challenging for most of the notions involved in our analogical hexagon. Let us face the challenge, trying to provide prototypical examples of the six corners of the hexagon.

3.1. Opposition

Finding a prototypical example of opposition seems impossible: we have seen that the three basic oppositions of the theory of the square of opposition are indeed incompatible two by two. Nevertheless since we are not afraid of contradiction we propose the following example:

Fig. 31 - PROTOTYPICAL EXAMPLE FOR OPPOSITION

This example makes sense if we consider that the opposition between men and women can vary according to the circumstances.

3.2. Difference

The corner of difference of our hexagon is a corner of the triangle of subcontrariety. Each corner of such a triangle is a disjunct union, one more time a real difficulty. Differences can be oppositions or analogies. We consider the following example:

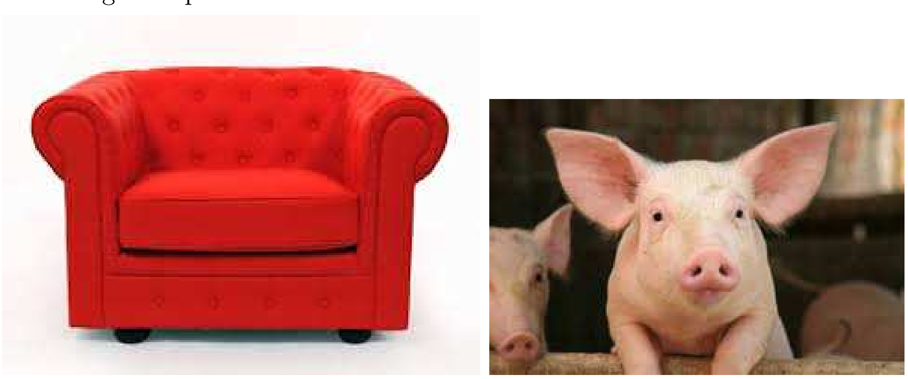

Fig. 32 - PROTOTYPICAL EXAMPLE FOR DIFFERENCE

This difference can be interpreted as an opposition or as an analogy, because it is not too much pushing on one or the other directions.

3.3. Identity

The case of identity is probably the easiest one, although there are some subtle problems that we will not discuss here (see [7]). We can choose the following example:

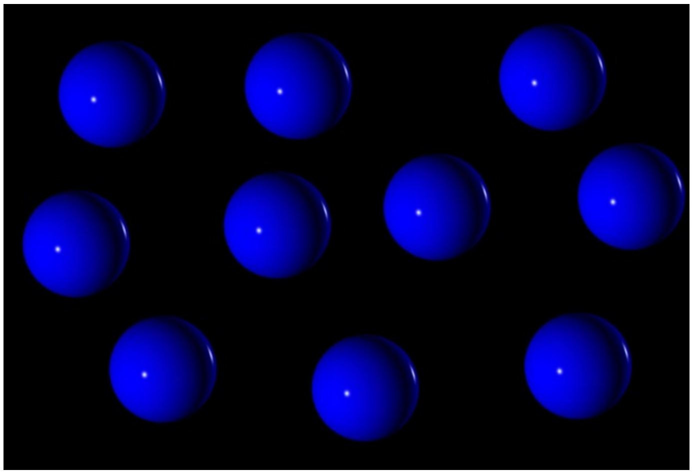

Fig. 33 - PROTOTYPICAL EXAMPLE FOR IDENTITY

3.4. Similarity

Similarity is also fairly easy. Below are two “cats”. They are similar not only because they are member of the same species but because of their furs.

Fig. 34 - - PROTOTYPICAL EXAMPLE FOR SIMILARITY

3.5. Analogy

Let's go now for our central character. Here is an interesting picture:

Fig. 35 - PROTOTYPICAL EXAMPLE FOR ANALOGY

It is showing the similarity between a bird and a plane. On the other hand we know that planes and birds are seriously different, in particular one is natural, the other artificial. Human beings have built plane certainly inspired by birds, they have transposed the configuration of birds into some machines. This analogy does not fall flat, it has taken human beings quite high in the sky.

3.6. Non-Analogy

To finish, we are facing again a pretty much difficult situation. How can we typically represent two things that can be seen as opposed or identical?

Fig. 36 - PROTOTYPICAL EXAMPLE FOR NON-ANALOGY

These two smurfs are identical, it is the same smurf duplicated and in opposed positions. In our example of identity above we also had some blue things which can be seen as the duplication of one and the same thing. The only difference is the location. Since these balls are spheric it is difficult to say that they are in different positions like two above smurfs, one looking to the right, the other to the left. These two smurfs are not just similar, they are identical, and their position is not just different but opposed.

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