What is an Axiom?

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Dedicated to Francisco Antônio Dória for his 75th birthday

Abstract

After some methodological considerations, we start by a general overview of the meaning of the notion of axiom. We then examine two kinds of axioms, on the one hand the Zurich axioms, on the other hand the axiom of choice. After that we discuss two trinities: definition/axiom/proof, axiom/proof/truth. We conclude by some remarks on the rise and fall of the axiomatic method. We end by a few recollections about Francisco Dória.

Keywords Axiom, Definition, Proof, Truth, Axiomatic Method

Hymne à l'axiome par le Baron de Chambourcy
Ce qui est évident n'est pas forcément vrai,
Ne nous emmène pas souvent très loin.
Ce n'est pas non plus nécessaire de faire des hypothèses très compliquées,
Suppositions douteuses qui ne feront qu'embrouiller notre esprit.
L'idée est plutôt de s'élever au-dessus du tumulte,
Avoir une perspective plus claire et plus générale,
Qui nous permettra de guider notre pensée afin d'atteindre la compréhension.
Resdecendre dans la vallée et d'en apprécier la beauté,
Sans tomber dans les pièges du détail.
Axiome, aile qui nous fait survoler les platitudes.

1 A Four-Dimensional Perspective

"Axiom" is first of all a word. This in fact can be said of many words, and this can be seen as rather tautologous or/and nonsensical. But "axiom" is emblematic in the sense that it is a word, by contrast for example to "God", "Cat" or "Banana", that has no real translation in other languages, only alphabetic adaptations.

Originally we have "ἀξίωμα" in Greek, a word with the direct transliteration "axioma" in Spanish, Portuguese or Interlingua; and with small variations: "assioma" in Italian, "Аксио́ма" in Russian, "axiome" in French, "Axiom" in German, English, Swedish, "Axiomo" in Ido, "Aksiyom" in Turkish.

It therefore works rather like a name of a country, a city, a river, a person, i.e. a proper name. Also similar to "axiom" are "logic" and "mathematics", which by chance have some direct connections with it.

However, despite the primacy of the word, we will not stay at this superficial level, but we will go deeper and deeper, higher and higher, trying to eradicate the root, going up to the fresh spring, opening some new perspectives.

In a previous paper we have promoted the pyramid of meaning [8]. This pyramid has a triangular basis: word, idea, thing (language / thought / reality). The top of the pyramid, supervising and synthetizing this triangle, we decided to call it "notion". The notional viewpoint reduces neither to language, nor to thought, nor to reality. It encompasses the three.

In the present paper we are dealing not only with the word "axiom", but also with the idea and reality attached to it, i.e. with the notion of axiom (no quotes!). To investigate a notion, we can go in four complementary directions:
• searching for a definition,
• understanding the relation of this notion with other notions,
• investigating the use of it,
• making a theory that will fix and clarify its meaning.

We will develop here this fourfold approach in a general perspective where these four aspects are intertwined.[1]

[1]This paper, as two other recent papers of mine ([9], [10]), has a strong methodological and pedagogical aspect: at the same time that I am dealing with a topic, I am trying to investigate how this can be done.

2 General Meaning

Before developing a critical discussion about a notion or elaborating a theory of it, it is always good to open our eyes to see how this notion has been conceived. This is important to avoid to be too idiosyncratic, on the other hand it is also important to avoid to sink in a descriptive ocean, an enumeration of all the ways a notion has been conceived, or can be thought in all possible worlds. To do that we will highlight here some definitions, quotes and associated words.

2.1 Definition

A definition can be found in a dictionary or in an encyclopedia. We have made a selection of 6 of them, 3 dictionaries and 3 encyclopedias.

Dictionary.com
1. a self-evident truth that requires no proof.
2. a universally accepted principle or rule.
3. *Logic*, *Mathematics*. a proposition that is assumed without proof for the sake of studying the consequences that follow from it.

Cambridge
1. formal
a statement or principle that is generally accepted to be true, but need not be so: *It is a widely held axiom that governments should not negotiate with terrorists.*
2. science, specialized
a formal statement or principle in mathematics, science, etc., from which other statements can be obtained: *Euclids axioms form the foundation of his system of geometry.*

Merriam-Webster
1. a statement accepted as true as the basis for argument or inference: *one of the axioms of the theory of evolution*
2. an established rule or principle or a self-evident truth: *no one gives what he does not have*
3. a maxim widely accepted on its intrinsic merit: *the axioms of wisdom*

Encyclopedia Britannica
Axiom, in logic, an indemonstrable first principle, rule, or maxim, that has found general acceptance or is thought worthy of common acceptance whether by virtue of a claim to intrinsic merit or on the basis of an appeal to self-evidence. An example would be: “Nothing can both be and not be at the same time and in the same respect.”

In Euclid’s Elements the first principles were listed in two categories, as postulates and as common notions. The former are principles of geometry and seem to have been thought of as required assumptions because their statement opened with “let there be demanded” (etestho). The common notions are evidently the same as what were termed “axioms” by Aristotle, who deemed axioms the first principles from which all demonstrative sciences must start; indeed Proclus, the last important Greek philosopher ("On the First Book of Euclid"), stated explicitly that the notion and axiom are synonymous. The principle distinguishing postulates from axioms, however, does not seem certain. Proclus debated various accounts of it, among them that postulates are peculiar to geometry whereas axioms are common either to all sciences that are concerned with quantity or to all sciences whatever.

In modern times, mathematicians have often used the words postulate and axiom as synonyms. Some recommend that the term axiom be reserved for the axioms of logic and postulate for those assumptions or first principles beyond the principles of logic by which a particular mathematical discipline is defined. Compare theorem.

Wikipedia

An axiom or postulate is a statement that is taken to be true, to serve as a premise or starting point for further reasoning and arguments. The word comes from the Greek axioma ($\alpha\xi\iota\omega\mu\alpha$) "that which is thought worthy or fit" or "that which commends itself as evident."

The term has subtle differences in definition when used in the context of different fields of study. As defined in classic philosophy, an axiom is a statement that is so evident or well-established, that it is accepted without controversy or question. As used in modern logic, an axiom is a premise or starting point for reasoning.

As used in mathematics, the term axiom is used in two related but distinguishable senses: "logical axioms" and "non-logical axioms". Logical axioms are usually statements that are taken to be true within the system of logic they define and are often shown in symbolic form (e.g., (A and B) implies A), while non-logical axioms (e.g., $a + b = b + a$) are actually substantive assertions about the elements of the domain of a specific mathematical theory (such as arithmetic).

When used in the latter sense, "axiom", "postulate", and "assumption" may be used interchangeably. In most cases, a non-logical axiom is simply a formal logical expression used in deduction to build a mathematical theory, and might or might not be self-evident in nature (e.g., parallel postulate in Euclidean geometry). To axiomatize a system of knowledge is to show that its claims can be derived from a small, well-understood set of sentences (the axioms), and there may be multiple ways to axiomatize a given mathematical domain.

Any axiom is a statement that serves as a starting point from which other statements are logically derived. Whether it is meaningful (and, if so, what it means) for an axiom to be "true" is a subject of debate in the philosophy of mathematics

The Columbia Electronic Encyclopedia

Axiom, in mathematics and logic, general statement accepted without proof as the basis for logically deducing other statements (theorems). Examples of axioms used widely in mathematics are those related to equality (e.g., "Two things equal to the same thing are equal to each other"; "If equals are added to equals, the sums are equal") and those related to operations (e.g., the associative law and the commutative law). A postulate, like an axiom, is a statement that is accepted without proof; however, it deals with specific subject matter (e.g., properties of geometrical figures) and thus is not so general as an axiom. It is sometimes said that an axiom or postulate is a self-evident statement, but the truth of the statement need not be evident and may in some cases even seem to contradict common sense. Moreover, a statement may be an axiom or postulate in one deductive system and may instead be derived from other statements in another system. A set of axioms on which a system is based is often wished to be independent; i.e., no one of its members can be deduced from any combination of the others. (Historically, the development of non-Euclidean geometry grew out of attempts to prove or disprove the independence of the parallel postulate of Euclid.) The axioms should also be consistent; i.e., it should not be possible to deduce contradictory statements from them. Completeness is another property sometimes mentioned in connection with a set of axioms; if the set is complete, then any true statement within the system described by the axioms may be deduced from them.

2.2 Quotes

Quotes is also an interesting tool which is generally not used systematically. This may be fine or/and funny to put one or two quotes at a beginning of a paper. But here what we are doing is different: we present a selection of quotes. The idea is to choose a set of quotes which are complementary (in a classical or Bohrian way) and exhaustive, i.e. encompass the main meanings of the notion, avoiding repetition. The authors of the quotes are important in the sense that they are considered as great and/or famous minds in a given field, and here also it is nice to have complementarity and exhaustivity. In this spirit we have selected the following nine quotes.

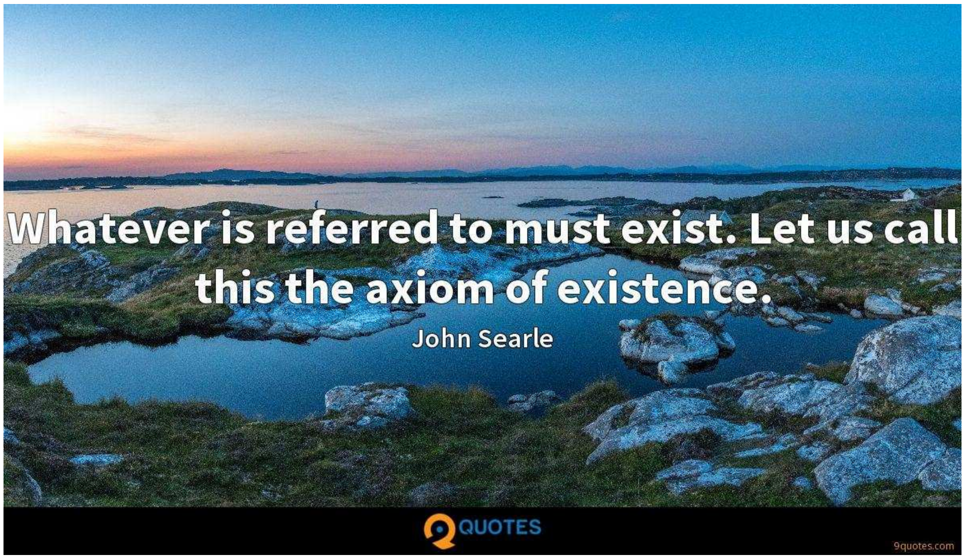
Henry Adams
I tell you the solemn truth, that the doctrine of the Trinity is not so difficult to accept for a working proposition as any one of the axioms of physics.

Alonzo Church
The only thing that might have annoyed some mathematicians was the presumption of assuming that maybe the axiom of choice could fail, and that we should look into contrary assumptions.

Tom Stoppard
I am not a mathematician, but I was awere that for centuries, mathematics was considered the queen of sciences because it claimed certianty. It was grounded on some fundamental certainties - axioms - that led to others.

Gustave Flaubert
There are neither good nor bad subjects. From the point of view of pure Art, you could almost establish it as an axiom that the subject is irrelevant, style itself being an absolute manner of seeing things.

L.Ron Hubbard
A science is something which is constructed from truth on workable axioms. There are 55 axioms in scientology which are very demonstrably true, and on these can be constructed a great deal.



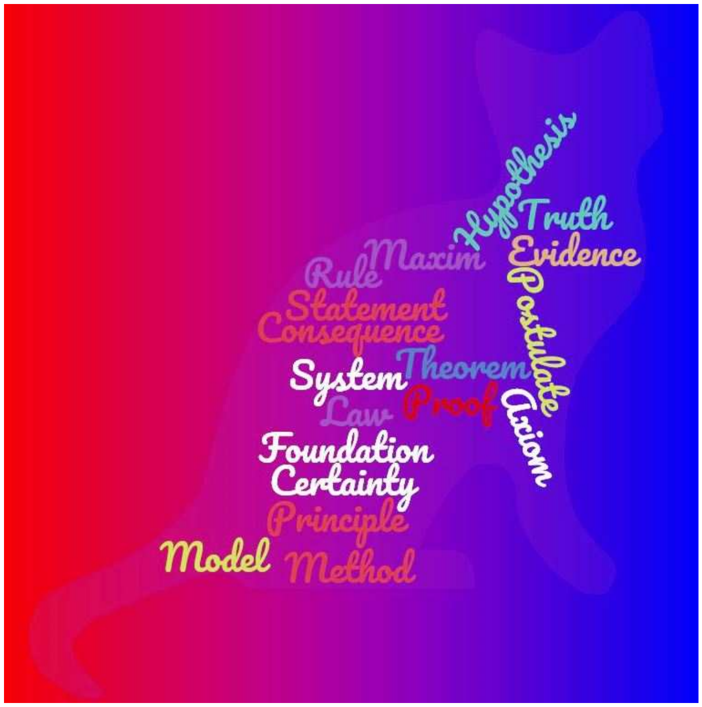
Albert Einstein
The grand aim of all science it to cover the greatest number of empirical facts by logical deduction from the smallest number of hypotheses of axioms.

Gottfried Leibniz
Finally there are simpl ideas of which no definition can be given, there are also axioms or postulates, or in word primary principles, which cannot be proved and have no need of proof.

Vincent McNabb
We shall not hold the dangerous axiom that 'truth is the best policy', because policy is but a means to an end; and truth is an end, not a means.

2.3 An Axiomatic Cloud

We are now ready to present a cloud of notions linked to the notion of axiom. This idea of cloud is related with the work I have done with Patrick Suppes at Stanford centered on the notion of *association* [26]. A cloud can be seen as a first step to establish a *network*. The notions in a cloud of a notion X are those which are most commonly associated to X, but the cloud is not articulated in the form of a hierarchy.



2.4 What an Axiom is Not

When examining a notion, it is also good to investigate opposite notions. At a linguistic level, this can be done by designing a cloud of antonyms. But we will not use this methodology here. We will give typical examples of statements which are not axioms. Here is a short list:
• *Tomatoes are not potatoes*
• $2 + 2 = 4$
• $\exists x \exists y \exists z (x < y) \wedge (y < z) \wedge \neg(x < z)$

Why are those statements not axioms? Here three reasons explaining the exclusion from the universe of axioms of these statements:
• A axiom is not a simple truth.
• An axiom is something that lead you somewhere: it should be useful as a principle of action, or deduction.
• An axiom should be pretty general.

The problem here is not the question of the distinction between *axiom* and *postulate*, or between an *absolute truth* and a *hypothesis*, or a *degree of generality.* About these questions and the evolution of the meaning of the notion of axiom, the reader may consult for example [20].

3 Two Axiomatic Examples

3.1 Zurich Axioms

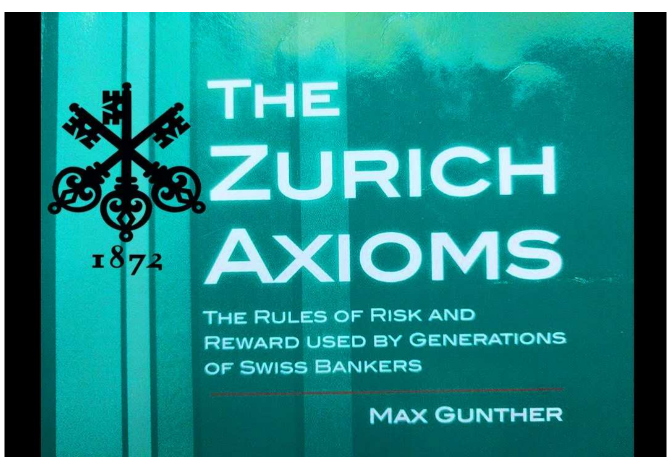
Before going directly to hardcore problems of logic and mathematics, let us deal with a lighter topic, banking. On the one hand for having a general perspective, on the other hand because it may keep some readers on the way. Our paper is not directed only for professional logicians and mathematicians but for a larger audience.

There is a book entitled *The Zurich Axioms*. It has been written by Max Gunther (1927-1998), the son of a famous Swiss banker, trying to explain how his father and other bankers succeeded to earn lots of money. Here is how he presents the book and the origin of the use of the word "axiom" in the title:

> The Swiss, observing all this, conclude that the sensible way to conduct one's life is not to shun risk, but to expose oneself to it deliberately. To join the game; to bet. But not in the caterpillars mindless way. To bet, instead, with care and thought. To bet in such a way that large gains are more likely than large losses. *To bet and win.*
>
> Can this be done? Indeed. There is a formula for doing it. Or perhaps formula is the wrong word, since it suggests mechanical actions and a lack of choice. A better word might be 'philosophy'. This formula or philosophy consists of twelve profound and mysterious rules of risk-taking called the Zurich Axioms.
>
> The list of rules evolved gradually. It grew shorter, sharper, tidier, and more useful as time went on. Nobody remembers who coined the term 'Zurich Axioms?', but that is the name by which the rules came to be known and are still known.

Here are some examples of these axioms:

- Always play for meaningful stakes.
- When the ship starts to sink, don't pray. Jump.
- Chaos is not dangerous until it begins to look orderly.



We have here *rules* or *principles* that are used to develop a certain activity. It can be an explicit command or a statement that will guide your action. We are therefore in a sphere similar to the one of a game or a religion. You may believe that if you obey the ten commandments you will go to heaven, or at least avoid to go to hell. At the level of gaming, these Zurich axioms are not like the rules you must obey to play e.g. chess, but some principles you may follow to win the game.

We can use the same generic word, "rule", for both cases to manifest the interplay between the two, but it also important to emphasize the distinction, or even the opposition: deontic rules vs. strategic rules (see [21]). Strategic rules are neither obvious nor mechanical. If it were the case everybody will be millionaire and chess champion. It is not like a cooking recipe: you follow the rules and the result is a nice cake!

An important feature of the Zurich axiom is that they are a synthesis of different ideas that gives a *direction*, an *orientation*. Axioms in logic and mathematics can also be seen in this way.

3.2 The Axiom of Choice

Let us now, before going to the general abstract notion of axiom in mathematics and logic, study a specific case of "real" axiom. Generality is nice, but it is also good to focus on a particular example, not a sample case, but a typical case.

The word "axiom" is widely known, its precise logico-mathematical meaning, if any, not so much. And if we ask for an example of axiom, it is not clear if something will promptly emerge, will be distinguished. There is of course the *axiom of parallel*, but it is rather known as a *postulate*. We will not choose it here as a guinea pig due to its long history. We will choose an axiom of modern times, *the axiom of choice*.

The name is famous, but few people know exactly what this axiom is, in particular for two reasons: on the one hand the name of this axiom is rather ambiguous and does not properly reflect its meaning, on the other hand there are many different equivalent versions of this axiom (see [23]), each having a different meaning. Only a hard extensionalist can claim there is only one meaning behind all the versions of this axiom.

The qualifier "choice" is highly ambiguous, because it gives the impression that it has to do with que difficult choice question we may face in every day life: how I will choose to go on the left rather on the right, decide to live in Paris rather than in New York, to become a dentist rather than a singer, etc.

The ambiguity of the meaning of this axiom turns the question of its truth or/and obviousness problematic. There can be two ways out: equivalence or derivation.

The equivalence way is to find a formulation of it which is more obvious than the other ones. One of the most famous formulations of the axiom of choice, Zorn's lemma (a partially ordered set containing upper bounds for every chain contains at least one maximal element), gives the impression that this axiom is rather artificial, far-fetched. On the other hand Edison Farah proved that general distributivity of conjunction relatively to disjunction is equivalent to the Axiom of Choice [15].[2] This seems rather "natural".

Another way out is to derive the axiom of choice from other more elementary axioms. This is what people have tried to do. However the fact that this axiom is independent of the axioms of the most famous set theory, *ZF*, shows that there is no easy way to derive this axiom from some basic axioms whose meaning and truth is rather simple and obvious.

Independently of its own meaning, one of the reasons to support this axion can be pragmatic: with it we can do lots of interesting things, we can prove many interesting results that cannot be proven without it. It can therefore be welcome from the viewpoint of a winning strategy. But among mathematicians there are some people who don't like this pragmatic view and this can be for quite different reasons. A "Platonist" may say that this axiom does not correspond to abstract reality of the mathematical universe. A "Constructivist" may argue that this axiom does not correspond to any constructive procedure.

[2]Edison Farah was the host of André Weil in São Paulo. Weil had the idea that this conjecture proven by Farah was false.

4 Two Trinities

4.1 Definition, Axiom, Proof

There is a famous *DAP* trilogy: definition, axiom, proof. It was promoted in particular by Blaise Pascal (1623-1662) as the basic methodology for mathematics. The ideas of Pascal [19] can be summarized in the following table:

PASCAL 8 RULES	
Rules for Definitions	Not to undertake to define any of the things so well-known of themselves that clearer terms cannot be had to explain them.
	Not to leave any terms that are at all obscure or ambiguous without definition.
	Not to employ in the definition of terms any words but such as are perfectly known or already explained.
Rules for Axioms	Not to omit any necessary principle without asking whether it is admitted, however clear and evident it may be.
	Not to demand, in axioms, any but things that are perfectly evident of themselves.
Rules for Proofs	Not to undertake to demonstrate any thing that is so evident of itself that nothing can be given that is clearer to prove it.
	To prove all propositions at all obscure, and to employ in their proof only very evident maxims or propositions already admitted or demonstrated.
	To always mentally substitute definitions in the place of things defined, in order not to be misled by the ambiguity of terms which have been restricted by definitions.

The *DAP* trilogy is the basis of the *axiomatic method*, despite the fact that this terminology just keeps explicit the notion of axiom. It is a bit weird, but we see a similar situation in the *holy trinity*, where the holy spirit predominates over the father and the son.

And, as in the case of Christianity, the relations between the three notions of the *DAP* trinity is quite complex. If we have, let us say, a group of axioms for beer and sausage, can't we say that these axioms are defining what beer and sausage are? Later on, on the basis of these definitional axioms, it is possible to introduce by definition, without further axioms, lots of different new entities, all kinds of goulash, by mixing these two ingredients. Related to this question is the question of primitivity. Is there a primordial soup of terms and axioms?

This point was seriously challenged in modern mathematics and logic. Alfred Tarski (1901-1983) was very much inspired by Blaise Pascal to develop the axiomatic method and he insisted that there were not absolute primitives (see [28], [27]). And in fact this relativity applies also to the distinction between proof and axiom, because a proof, yes, can be considered as an axiom and vice versa!

Confusions regarding the *DAP* trinity show up when talking about set theory as a basis for all mathematics. It is true that axiomatic set theory, let us say the most famous one, *ZF*, permits to define many concepts of mathematics, like for example the concept of group. But this does not mean that the axioms of group are a consequence of *ZF* axioms (see [2]). We cannot say that set theory allows to define everything in the universe and that through it we can axiomatize the whole universe. This would be exaggerated in many senses.

Also it is exaggerated to say that we can apply the axiomatic method to everything. In philosophy, Spinoza (1632-1677) was the only one, who really tried to do so. But Pascal, his contemporary, explicitly said that he did not believe that this method applies to everything, and that in particular it makes no sense to try to define the notion of human being.

4.2 Axiom, Proof, Truth

The axiomatic method was improved in many ways in modern logic and modern mathematics. Lots of new distinctions and concepts appeared. There is of course the question of *formalization*. The way to express propositions was systematically developed in more details, leading in particular to the language of first-order logic.

An important distinction is between proof theory and model theory. These two theories give two completely different visions of what an axiom is. This is why the completeness theorem which links the two is so spectacular.

The proof-theoretical vision is close to the traditional one: using some rules, we deduce step by step some theorems from axioms or/and definitions. The main difference between the "tradition" and modern times is that was developed a systematic theory, called *proof theory*. In particular in this theory the "steps" were made clearer and more explicit.

The model-theoretical vision is fairly new because it establishes the relation between axioms and "reality". What kind of reality? That's the question! This reality is structural: we have some structures, that can be conceived from a set-theoretical point of view and that can correspond to the real world in its various aspects. If a structure *STR* obeys a group of axioms *AXI*, we say that

STR is a model of *AXI*, and that *AXI* are true in *STR*. The heart of model theory is to articulate truth in a structure, and it took several decades before this was made clear by Tarski (cf. [18]).

A group of axioms *AXI* may have a variety of models quite different from each other. For example if we consider the basic axioms *ORD* for the notion of order, antisymmetry and transitivity, we have a good variety of heterogeneous models: partial order vs. linear order, discrete vs. dense order, with end points vs. without end points, and so on. We can nevertheless say that we have *axiomatized the notion of order*. But that is different from *axiomatizing the reality of a given structure*, let's say the structure of natural numbers, *STN*. To axiomatize *STN* means to find a group of axioms *AXN* such that the only model of *AXN* is *STN*.

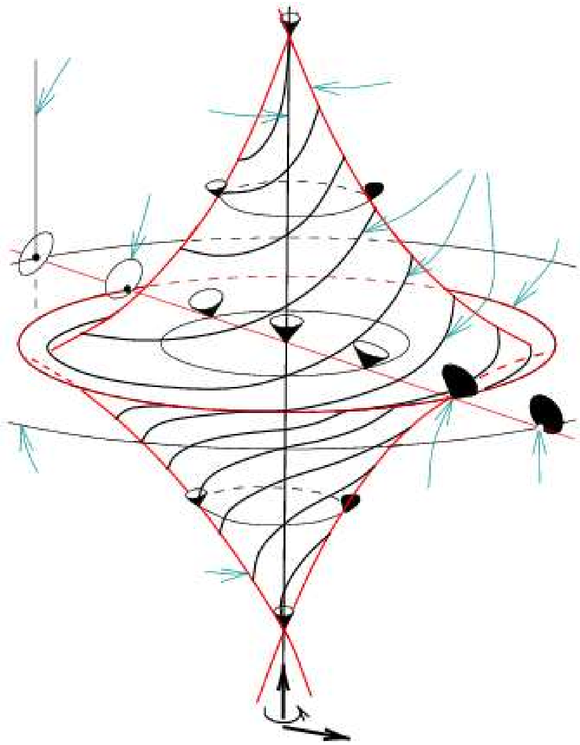
Thoraf Skolem (1887-1963) proved an important metatheorem [25] about non-standard models of arithmetic, using the compactness theorem. He showed that we cannot axiomatize *STN* model-theoretically: given a set of axioms for *STN*, it is always possible to find a model which is radically different, in the sense that there are some non-standard numbers, coming after all the standard numbers.

On the other hand Kurt Gödel (1906-1978) with his incompleteness metatheorem [16] showed that it is not possible to proof-theoretically axiomatize the natural numbers. Considering any axioms of a proof-system for *STM*, there will be a proposition such that neither this proposition, nor its negation can be deduced from them, the famous Gödel's proposition inspired by the liar paradox.

In both cases these results are related to first-order logic and recursion. "Recursive" is a technical term to talk about mechanization or computability. Modern logicians were able to precisely characterize through recursion theory these informal notions. We will not here enter in details. But what is important is the set of axioms to be sizable. If we take all the propositions true in *STN*, this is obviously an axiomatization of *STN*. The other aspect of recursion is also about "sizability": if we consider that a proof is any sequence of propositions, then everything is proof-theoretically axiomatizable.

As we have pointed out, the word "axiom" in modern logic is used in two very different ways. The completeness theorem establishes a bridge between the two but not equate them. It is very important to keep in mind this distinction. To use two different words would be a bit artificial, but when necessary, in case of ambiguity, it is important to specify what we are talking about.

How to export the axiomatic method outside of mathematics? One may want to find some axioms describing the structure of the universe, in the sense that the universe is the only model of these axioms. And then we can deduce all the truths about the universe from these axioms. We see here the intertwining between the proof-theoretical and the model-theoretical notions of axiom.



Dória together with Newton da Costa proved the incompleteness of classical mechanics and other physical theories (see [14] and [13]). And Gödel has a famous result showing that Einstein's theory of relativity admits non rotating universe as models [17]. It is important to clearly distinguish two aspects of this result. The first is than Einstein's theory does not axiomatize the universe, rotating or not, because there are radically different models of this theory. The second point is to know if it is natural or not to have rotating universes and how to eliminate them if considered as parasitic.

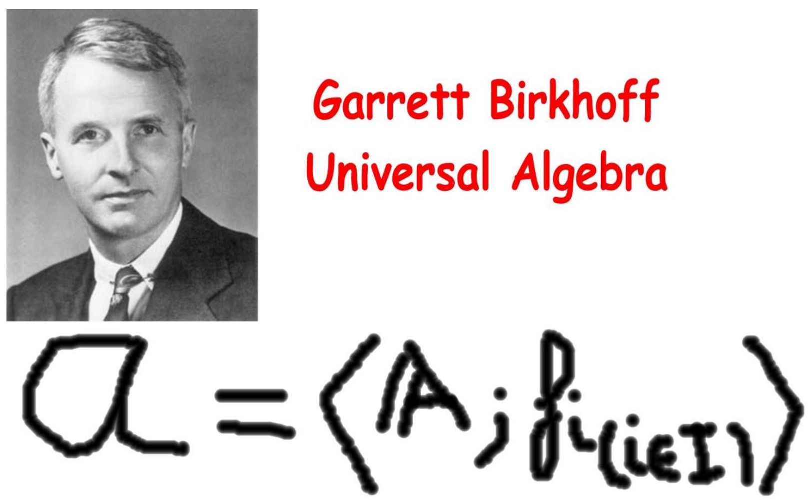
5 The Rise and Fall of the Axiomatic Method

The axiomatic method is not anymore at the center of the stage and probably will never be again. The golden age of the axiomatic method was the first half of the 20th century. The axiomatic method was strongly promoted in modern logic and modern mathematics, through in particular the work of Hilbert and Bourbaki. And there was also the will to apply it to all sciences: biology, physics, economics, etc.

We have now a clear idea that it is not possible to find a single axiom, or a group of axioms, from which everything can be deduced, computationally or not, or which characterizes, describes the universe of sets or monkeys, or the universe in its entirety. This impossibility does not prevent to use this method in an interesting way, and not only in logic and mathematics. We can also fruitfully use it in any field, or to develop the understanding of fundamental notions, such as for example causality.

It is important to still promote and use the axiomatic method, we cannot go back to prehistoric time where this method was not known. The axiomatic method, born in Ancient Greece, is certainly part of the seven wonders of thought.

But there are other wonders. One of them is conceptualization. They can go hand to hand, they are not opposed, rivals, enemy sisters, like Kyana and Keyla. They rather are in a subcontrary opposition, than a contrary or contradictory opposition (about these notions, see e.g. [7]). Moreover conceptualization can go alone by herself and need not be subordinated to axiomatization.



This is was clearly shown by the work by Garret Birkhoff on *universal algebra* [12]. He was able to conceptualize notions like those of subalgebra and morphism, without any axioms. It is indeed easier to conceptualize these notions with no axioms, considering an algebra just as a family of operators on a set, obeying, yes, no axioms! Farewell to universal algebra in the sense of Sylvester and Whitehead.[3]

Inspired by Birkhoff, I decided to develop *universal logic* along the same conceptual line of thought, considering a logical structure, as a structure obeying no axioms [1], by contrast to Tarski's general approach to the metalogic theory of consequence operator [3], based on three axioms [4].

Tarski's approach is already breaking the paradigm of *laws of thought*, in particular because it is an abstract framework with no logical operators. At best Tarski's axioms can be considered as *laws of consequence*. But although

[3]Category theory is also going in this direction, see [24] and [22].

Tarski had a the very general perspective of a general theory of science, he was focusing on deductive sciences, cf. the expression "Methodology of deductive sciences", which is part of the title of his introductory book of logic [28] and of the title of the series of events he launched, CLMPS: *Congress of Logic, Methodology and Philosophy of Sciences*.

If we go to empirical sciences there are good reasons to reject the second Tarskian axiom, the basis of a class of logic has been nicely negatively baptized by John McCarthy (1927-2011): *non-monotonic logics*. There are also good reasons to reject the two other axioms, reflexivity and transitivity, both for theoretical and practical reasons. We can promote what I have called *axiomatic emptiness* [6].

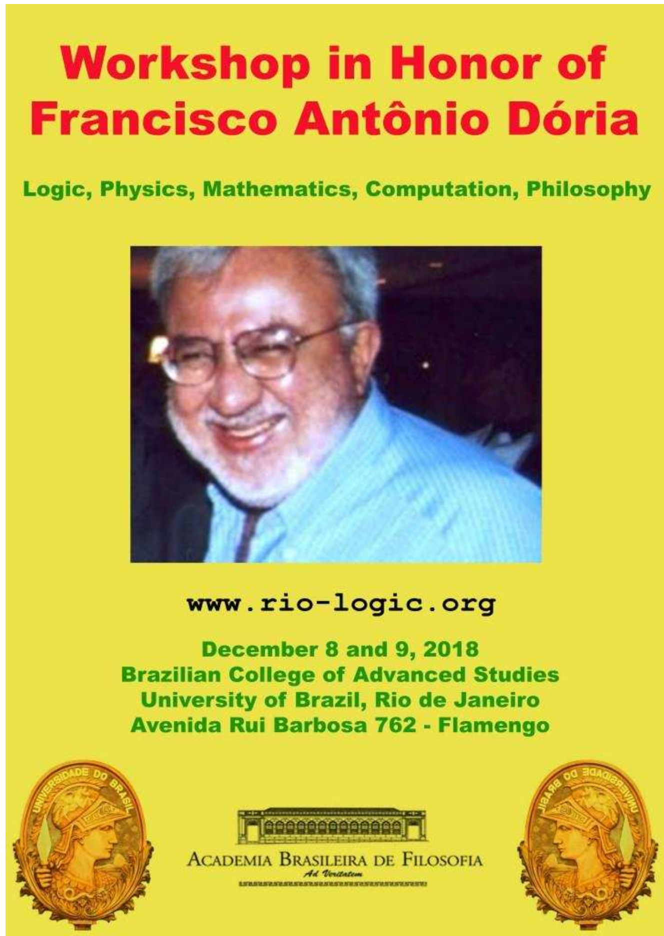
If we want to develop a very general theory of reasoning, in an abstract and mathematical way, it is good to just start with concepts. Then we can fruitfully use and apply these concepts in some specific cases where axioms will show up. This methodology is neither in favor of monism or pluralism. The idea is to encompass and systematize the various systems of logic in one unified general approach which is itself not one system, based on some axioms, describing one way of reasoning.

6 Dedication and Personal Recollections

I have known Francisco Antônio Dória since many years. I don't remember exactly when and where I met him for the first time, probably in 1991 or 1992 in São Paulo. I came for the first time to Brazil in August 1992 to work with Newton da Costa at the University of São Paulo and Dória at this time was one of his main collaborators working on physical matters. Dória was living in Rio de Janeiro but used to come to São Paulo to work with da Costa.

Dória was member of my PhD jury at the University of São Paulo and we edited together a special issue of *Logique et Analyse* [11]. I moved to Rio de Janeiro in 1995 and I regularly met him in Rio or in Petrópolis, where he had a country house, discussing about all kinds of topics.

In 2000 I went for two years at Stanford University to work with Patrick Suppes. Dória had been there before and one of his PhD students, the editor of the present volume, Acacio de Barros, was at the time a main collaborator of Suppes and helped me to settle in the Farm. Going back to Brazil Acacio sold me his horse, called *Voyage*.



Acacio and I organized an event in honor of Dória December 8 and 9, 2018 at the *Brazilian College of Advanced Studies* of the *Federal University of Rio de Janeiro*, sponsored by the *Brazilian Academy of Philosophy* of which he is a member (Chair 21). As illustrated by the above poster, Dória is a true polymath.

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