

# Cats that are not cats

## Towards a natural philosophy of paraconsistency

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### Abstract

In this paper we present a conceptual framework that can philosophically support the idea of paraconsistent negation. After a short introduction explaining the perspective of the paper, we discuss in a first part in which sense classical negation is artificial. In a second part we present the idea of paracomplete negation and explain why it is more natural. Basic ideas for the development of paraconsistent negation are then introduced in a third part. The fourth part is a short conclusion and an assessment of the situation with open perspectives for future work. In the fifth and final part we present some recollections about our joint work with John Woods and explain the choice of the topic of paraconsistency to honor him. We wander in the labyrinth of thought guided by Candy and Cats.



*Une chatte qui n'est pas une chatte, en voilà une belle affaire!  
Une fente dans le mur de notre raison, un terrible gouffre pour la pensée ...  
Ou alors ne serait-ce qu'une mascotte propice à la masturbation intellectuelle?  
Telle la Poudre de Perlimpinpin, le Sexe des Anges,  
ou tout autre Soufisme de Byzance  
Baron de Chambourcy*

## 0 Paraconsistent Logical Systems and their Non-Philosophy

There are many systems of paraconsistent logic.<sup>1</sup> A paraconsistent *logic* can be defined as a logic in which there is a paraconsistent negation. A paraconsistent *negation* is a negation which is “non-explosive”, i.e. such that from  $p$  and  $\neg p$  it is not possible to deduce everything. Explosion is a typical feature of classical negation and, by contrast, its rejection is the key to paraconsistent negation (see [7]).

One attack against paraconsistent negations can be the claim that such negations are not negations (see [54], [10], [8]). But the development of paraconsistent systems of logic has shown that a paraconsistent negation can have many logical or/and metalogical properties of classical negation, more than some halves (due to incompatibilities this is not a linear order, there is not one strongest paraconsistent negation, but some maximal ones). This is a kind of quantitative argument in favor of the concept of paraconsistent negation. The qualitative aspect is more complicated, in particular because it should rely on some philosophical ideas and/or intuitions. And up to now the philosophy of paraconsistency is quite unsatisfactory, not to say deserted.



There is a trend which is related to Hegel, Marx and Mao, according to which contradiction is the basis of everything, the heart of thought and/or reality. The word *dialectic* has been used in this context.<sup>2</sup> Not to be confused with

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<sup>1</sup>There is not yet any textbook on paraconsistent logic. The introductory book by Manuel Bremer [41] is a kind of survey, although valuable it is rather incomplete and out of date. There are technical books dedicated to some given classes of systems of paraconsistent logics (see [3], [47], [43]), with very few philosophical discussions. More philosophical books, but with few technicalities, are the one by Graham Priest [53] and the one by John Woods himself [57]. And there are collections of papers corresponding to the five *World Congresses on Paraconsistency*: Ghent, 1997 [4]; Juquehy, 2000 [42]; Toulouse, 2003 [30]; Melbourne, 2008 [55]; Kolkata 2014 [32].

<sup>2</sup>Originally the word “dialectic” was used by Plato in a quite different way. Nowadays if you use this word, let’s say in a supermarket, one may think you are a Marxist, unless she

*dialetheism*, a word created only recently in the context of paraconsistent logic, for promoting a theory according to which there are some so-called *true contradictions*. It is not clear if the similarity of the two words *dialectic* and *dialetheism* is purely accidental. Some logicians working in paraconsistent logic in the 1970s were sympathetic with Marxism and the very expression “true contradiction” is itself in harmony with an ideology according to which contradiction is a kind of Goddess. Paraconsistent logic has been used in both contexts to back up the notion of contradiction. But in both cases there is much confusion about the very concept of contradiction and loose connections between the philosophical ideas and the formal systems.

For example, as we have recently pointed out (see [17]), on the one hand Graham Priest gives as an example of true contradiction the liar proposition of the liar paradox. This is a very special proposition, stated by a given man, Epimenides, in given location, Crete, at a given time, 555 B.C. And on the other hand he promotes a paraconsistent logical system, known as *The Logic of Paradox* (frequently abbreviated as *LP*) [52],<sup>3</sup> where all propositions are paraconsistent, not therefore making a distinction between the liar proposition and other propositions. Poor Eubulides must be rolling over in his grave.



The looseness between formal systems of logic and philosophical ideas is in fact quite typical of modern logic. A symbolic example, that we have recently discussed (cf. [18] and [23]), is the question of possibility and modal logic. It is important to try to close the gap, to have a more dialectical relation between the two. In this paper we present an intuitive ground for paraconsistent negation and give some hints of how it can be formally developed matching this natural philosophy.

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knows a bit of history of philosophy, in the same way that if you wear a Swastika, one may think you are a Nazi unless you are in India.

<sup>3</sup>This is a new name for a logic which was developed by the Argentinian mathematician Florencio Gonzalez Asenjo (1926-2013) under the name *Logic of Antinomies*. His first publications on the topic were in Spanish in the 1950s, followed by publications in English in the 1960s and the 1970s (see [1], [2]).

# 1 The Mysterious and Artificial Classical Non-Cat

Classical negation appears at first easy and natural, but it is just because we are used to it, like many things. Consider for example the number zero: nowadays nobody thinks it is something very strange. Everybody knows how to add 0 to another number, the result is this other number, and when multiplying a number by 0, the result is 0. These operations are quite easy to perform, we are able to compute these results faster than any super computer.

If Candy<sup>4</sup> says that she has 0 cats in her house, although it may look a bit strange or at best poetic, we understand that it means that there are no cats in her house, by contrast with Candy saying she has 3 cats in her house. And if we say to Candy that we will add one 0 to her salary she will immediately understand what it means.

So 0 looks natural, and indeed it is part of the family of *natural* numbers. But it was not the case at the start. It took many years before 0 being adopted in this family. At first people were not even able to think about it. The same with classical negation. The two were symbolically unified in modern mathematics by André Weil, father of:



The empty set is the set having no elements, or in a more pompous way, the collection of objects not identical to themselves. These are two extensionally equivalent definitions based in both cases on classical negation.

A non-cat is an animal much more mysterious than a sphinx, a centaur, a mermaid, the Loch Ness Monster, Charybdis, Scylla and Cætera. What kind of animal is she? Maybe she is not even an animal! A strange thing that the more creative minds, from Homer to Isaac Asimov, via Leonardo da Vinci, Lewis Carroll and Borges, were not even been able to imagine. Candy may think that such a monster does not make sense, that she does not want to have such kind of beast in her menagerie.

A non-cat is something which is *not* a cat. But what is not a cat? To be a cat is to be someone or at least something. But is being a non-cat, to be anything at all? A dog is not a cat, a stone is not a cat, hunger is not a cat, the number 4 is not a cat, “cat” is not a cat. Millions of things fell under the non-cat umbrella and these things are incredibly heterogeneous. Things which are non-cats are extremely varied both in quantity and quality. Like the contents of a garbage can or, to be more glamorous, like the scenery of Ali Baba’s cave.

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<sup>4</sup>I chose this name for the decorative character of this story, having recently visited the *Kingdom of Candia*.



DIFFICULT TO SINGLE OUT THE CLASSICAL NON-CAT IN ONE SOLE IMAGE!

The notion of non-cat is very abstract. It is something that seems impossible to grasp, capture or define. It can be pictured only symbolically, by putting a cross on the picture of a cat (and linguistically by putting “non” in front of “cat”). But thought is very powerful and can go beyond what is imaginable (see [19]). Classical negation is able to catch all the non-cats, putting them together, unifying this incoherent multiplicity. Classical negation is a jump into abstraction, it is a way for thought to capture the unknown from the known. A non-cat is an object of thought. Of logical thought.



THE CLASSICAL NON-CAT: A CREATURE FROM THE KINGDOM OF LOGIC

According to classical negation, all we know about the concept of non-cat is that something cannot be at the same time a cat and a non-cat and everything is either a cat or a non-cat. Cat and non-cat are *contradictory* concepts, following the theory of the square of opposition, distinguishing three notions of opposition: *contradiction*, *contrariety*, *subcontrariety*.<sup>5</sup> Classical negation is a very powerful tool that drives us from a thing to something which is almost nothing, from truth to falsity, from meaning to nonsense.

It is difficult to trace the origin of classical negation for various reasons. One of them is that it is something that it not that simple to identify or/and define. Classical negation of modern classical propositional logic is certainly not the same as any classical concept of negation, if we mean by “classical” something related to Greek antiquity: Athena, Aristophanes or even Aristotle. In particular because the framework is different. In modern logic negation is considered as a *connective*, which can be distinctly seen syntactically, as an operator building a proposition from another one, and semantically, its semantics being given by the following truth table

$p$	$\neg p$
V	F
F	V

meaning that the a proposition is true iff only its negation is false.

Classical negation is dichotomic. It can be seen as an abstraction of empirical dichotomies. The school of Pythagoras promoted the notion of dichotomies (cf. the Pythagorean table of opposites). Dichotomies can be more or less artificial. Both terms of a dichotomy can be natural, such as day and night, male and female, beautiful and ugly. When we go to a dichotomy like finite and infinite, this is more artificial, an artificiality which is syntactically reflected by the construction of the word “infinite” from the word ”finite” using a prefix. Now if we systematically apply to any concept the particle “non” to it with (or without) a hyphen, we reach a real artificial level: non-red, non-cat, non-sense.

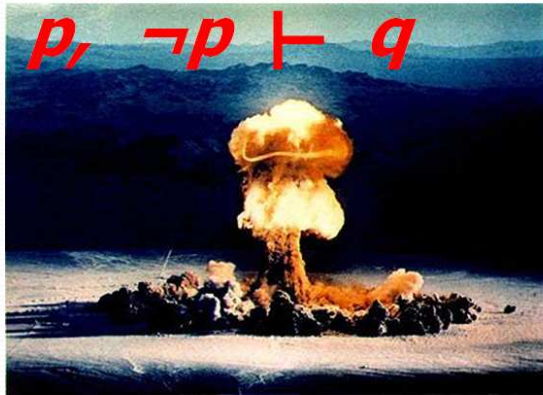
According to the mythology, Pythagoreans had the idea that everything was (or could be explained by) numbers, considered as positive natural numbers or pairs of natural numbers (i.e. rational numbers). Then by the use of classical negation, through the reduction to the absurd, they proved that square root

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<sup>5</sup>For a basic introduction to this theory and the frequent confusion between the two first notions, see [20], for recent collection of papers on this topic, see [39], [38], [37], [29], [34], [36], [31], [16].

of two is not rational. But what does it mean to say that  $\sqrt{2}$  is non-rational, or *irrational*, as Candy would say? The characterization of a number as non-rational does not tell us anything about its inner nature, if any. Nevertheless this non-rational number corresponds to reality, the diagonal of a square (another nice example is  $\pi$  and the circle, the famous apple pie).

Classical negation drove us to irrationality. She was able to break down an ideology, based on beliefs and intuitions, according to which natural numbers were the key to everything. According to the legend, the proof of irrationality of  $\sqrt{2}$  took place in a boat full of Pythagoreans, and the one who performed the proof was tossed overboard. But irrationality made her way. Mathematics was mainly developed with classical negation and with mathematics human beings were able to go to the moon. So we have to think twice before tossing classical negation overboard.



## Classical Negation is Explosive and Artificial



We can say that classical negation is artificial, if we define something artificial as something produced by humans. But what is artificial is not necessarily bad or wrong. A piano, a pizza, a plane are all artificial devices, and so is democracy, poetry and science. Classical negation is a very powerful engine, product of the two nipples of science: abstraction and generalization. It is an amazing tool also able to make us laugh (see [25]). We may nevertheless want to look for something smoother and subtler, that makes us gently smile rather than bursting out laughing like a crazy horse.

This will no replace classical negation but will be a good company to her. In this perspective we will introduce into the Kingdom of Logic, various non-classical non-cats with whom the classical non-cat will be happy to play with. And Candy will certainly be glad to have more than only one non-cat.

## 2 The Friendly and Familiar Paracomplete Non-Cat

We will not go straight to the point, to the paraconsistent non-cat. We will make a detour through the jungle of thought visiting another wild animal, the paracomplete non-cat. Beside explosion, another famous feature of classical negation is the so-called *principle of excluded middle*, according to which  $p$  and  $\neg p$  cannot both be false. To use the square terminology: they are *contraries*. If we go at the level of concepts this means that for example a number is either odd or non-odd, it cannot be neither odd, nor non-odd. If we consider natural numbers this makes sense, but now if we ask Candy if  $\sqrt{2}$  is odd or not, she may reply neither yes nor no, nor even maybe. And if we ask the question about her cat, she will find this even odder, not even trying to put the animal in the balance of parity.

So there is a good reason to develop an alternative notion of negation according to which a thing can be neither such, nor non-such. This good reason is the context, or to use a more spacy notion, the circumstances. It makes sense for Candy to say that Goofy is a non-cat, because he is an animal which is not a cat. But does it make sense for Candy to say that the Eiffel Tower is a non-cat? It is obviously not a cat, but it is also quite natural to say that it is not a non-cat. Cattiness does not apply to such an object, in the same way that redness does not apply to the number 4. It is neither red nor non-red, although we can consider a red “4”, but this is a kind of red-herring, a trap in which Candy will not fall.

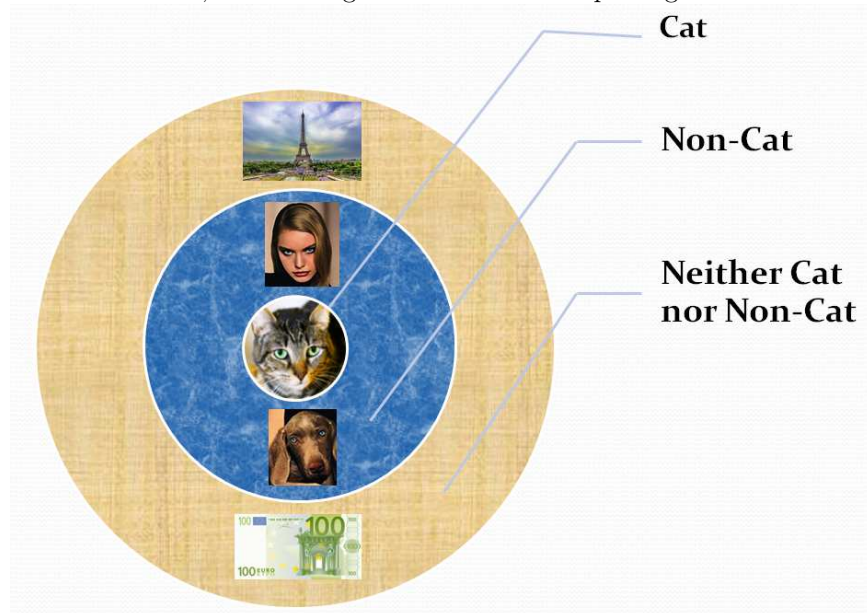


ON THE LEFT TWO PARACOMplete NON-CATS  
ON THE RIGHT SOMETHING WHICH IS NEITHER A CAT, NOR A NON-CAT



Among things that are not cats, a paracomplete negation allows to make a distinction between things which are not cats but *similar* to cats, and things which are not cats at all, i.e. completely *different* from cats. Vive la différence! (see [26]) Candy may say: that's nice, but where exactly to place the border wall (and at what cost)? A possible answer is that all animals that are not cats: mouses, snakes, monkeys, women, etc., are paracomplete non-cats. And that things like stones, clouds, numbers are not non-cats. But what about flowers, bananas or mushrooms?

Candy should not be afraid by weird borderline cases. Every case is a case and criminals will be prosecuted. But there is the general idea of a halo of circumstances surrounding any concept. For most concepts, we can consider: its positive part, a negative halo, and then a zone of incompleteness, which is outside of the halo, surrounding the halo and encompassing all the rest.



CIRCLING PARACOMPLETE NEGATION

Candy may wonder in which sense such a halo is a negation, since we have something here not obeying one central feature of classical negation. Let us first note that the rejection of the excluded middle is compatible with explosion, in the sense that an operator can be explosive and not obeying the principle of excluded middle. Also, as it is well-known from intuitionistic logic, the weak form of the reduction to the absurd is compatible with the elimination of the excluded middle (see [5] for details). If we define a paracomplete negation as a unary operator not obeying the principle of excluded middle, intuitionistic negation is such a paracomplete negation. It is a particular case. The philosophical “halogenic” notion of paracomplete negation we are promoting here is more general.<sup>6</sup>

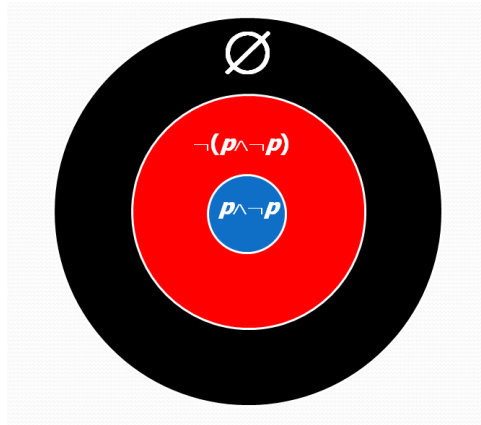
<sup>6</sup>The general idea and terminology of paracompleteness was introduced by Newton da

Candy may also wonder if any Thing is halogenic, in the sense of being surrounded by paraconcomplete non-Things. First of all let us point out that up to now we have rather dealt with concepts than propositions. But a concept can easily be embedded into a proposition. Instead of talking about the non-cat Goofy, we can talk about the proposition corresponding to the sentence *Goofy is a non-cat*. What is more difficult is the other way round, to reduce propositions to concepts, for example in case of a proposition mirroring an event. We will not enter too much in details here, rather focusing on the conceptual approach. But what we are saying about non-halogenic things applies as well to propositions as conceptual, eventual or grammatical.

If we have a proposition which is an antilogy, i.e. which is false under any circumstances, then we may have good reasons to think that it is not halogenic: its negation cannot also be false, since there is here no shadow of a doubt. We can agree with the young Wittgenstein [51] according to which such a proposition is meaningless. Therefore it makes sense for its negation also to be meaningless, being always true, leaving no incomplete zone.

The most famous antilogy in classical logic is  $p \wedge \neg p$ . It is considered as a *contradiction*. In the context of a paracomplete logic, this proposition can still be an antilogy, but it is in general not anymore a contradiction, since  $p$  and  $\neg p$  can both be false.

If  $p \wedge \neg p$  is an antilogy, we can consider that it is not halogenic and that  $p \wedge \neg p$  and  $\neg(p \wedge \neg p)$  cannot both be false (and the same for any antilogy). In this case  $(p \wedge \neg p) \wedge \neg(p \wedge \neg p)$  is a contradiction. And also  $(p \wedge \neg p) \vee \neg(p \wedge \neg p)$  is a tautology.



#### CIRCLING AN ANTILOGY WITH A PARACOMPLETE NEGATION

The proposition  $p$  can correspond for example to the *The Eiffel Tower is a cat*. Following the idea of a paracomplete negation presented here, both propositions *The Eiffel Tower is a cat* and *The Eiffel Tower is not a cat* can be false, but they cannot be true together. *The Eiffel Tower is a cat and is not a cat* is always false and therefore its negation is true. This proposition does not fall into the incomplete zone.

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Costa, as a dual of paraconsistency, see in particular [49] and [45] for a particular system.

### 3 The Wild and Fictitious Paraconsistent Non-Cat

Does it make sense to say that something is a cat and not a cat? Can we find a concept of non-cat having an intersection with the concept of cat? That's not so difficult or artificial as Candy may imagine.



A FAMOUS STANDARD CAT: LARRY  
CHIEF MOUSER TO THE UK CABINET OFFICE, SINCE 2011

The concept of cat is in fact two-dimensional. On the one hand we have the domestic cats, the purring cats, the standard cats, such as Kitty, Pussy and Larry. On the other hand we have the wild cats, the roaring cats, such as tigers, lions and panthers. The whole cat family is called the *Felidae*. We can say that a tiger is a cat and is not a cat, because it is not a domestic cat but it is a feline. It is a big cat, a *Cat*.



AN ANONYMOUS CAT WHICH IS NOT A CAT: A SIBERIAN TIGER

Candy may say that we are playing with words, but language is nothing else than a word game. We have to play it right. The same with thought. Here we are not only using the fact that in English we have the same word for a concept and a concept included in it. This is also the case in German with *Katzen* but not for example in French: *un Tigre n'est pas un "chat"*. Nevertheless we can also say in French that a tiger is a cat and not a cat: *Un tigre est un chat qui n'est pas un chat*. Et oui damoiselle! Thought game does not reduce to word game.

The important thing is that we have a concept, the felidae concept, and a hypoconcept, the concept of domestic cat. In some sense domestic cats are the cats that are more cats among all felidae, more cats than tigers or other outdoor cats (despite the fact that outdoor cats are natural cats by contrast to domestic cats which are artificial cats, living close to human beings, in their domicile, near the fire place). We can say that domestic cats are at the heart of the cat family (*Home is where the heart is* as Candy likes to sing). Domestic cat is a heartfelt or essential hypoconcept of the concept of cat.

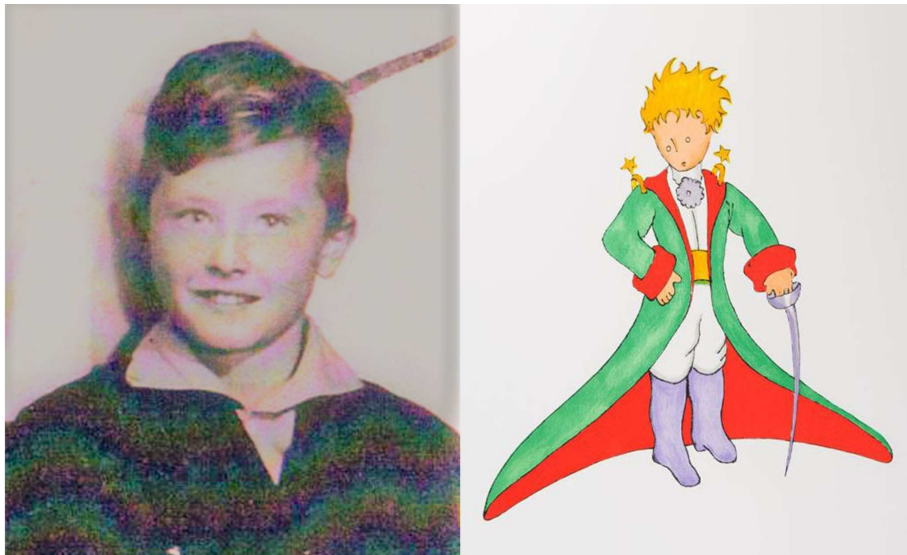
Given a heartfelt hypoconcept of a concept, we can call the zone between the concept and the heartfelt hypoconcept, the *inrounding of the concept*. It also can be called the *inconsistent zone*. This is the zone that is mastered by paraconsistent negation. Tigers, lions, panthers are living in this paraconsistent jungle, in the inrounding of the cat concept. They are at the same time cats and non-cats. Are all these animals, *contradictory* animals? Not at all! Because for two concepts to be contradictory they have to exclude each other (and together be exhaustive), according to the proper square definition. Et oui damoiseau!



AN ILLUSTRIOUS CAT WHICH IS NOT A CAT: FELIX

Our theory obviously applies to other species like Potami, centered around the hyopotamus, but Candy may ask: what about kettles, skirts and other jewels? Even without being an essential fundamentalist it is easy to find for most of the concepts a heartfelt hypoconcept and consequently an inconsistent zone. Let's consider Felix the Cat: he is not a Big Cat, but nevertheless can be considered as not being in the heart of the concept of cat, just because he is not real. For this reason he is a cat and a non-cat.

The same can be said about many fictitious creatures but also, the other way round, about their sources of inspiration. Let's take the example of one of the most famous fictitious creatures *The Little Prince* of Antoine de Saint-Exupéry. A basic inspiration for this character was Thomas De Koninck that Saint-Ex met in Québec one year before writing the story: "In 1942, Antoine de Saint-Exupéry visited friends in Québec City. While there, he met a precocious eight-year-old boy with curly blonde hair named Thomas whose inquisitive nature is said to have been the inspiration for *Le Petit Prince*. Thomas De Koninck is now a philosopher and professor at Laval University." [48]



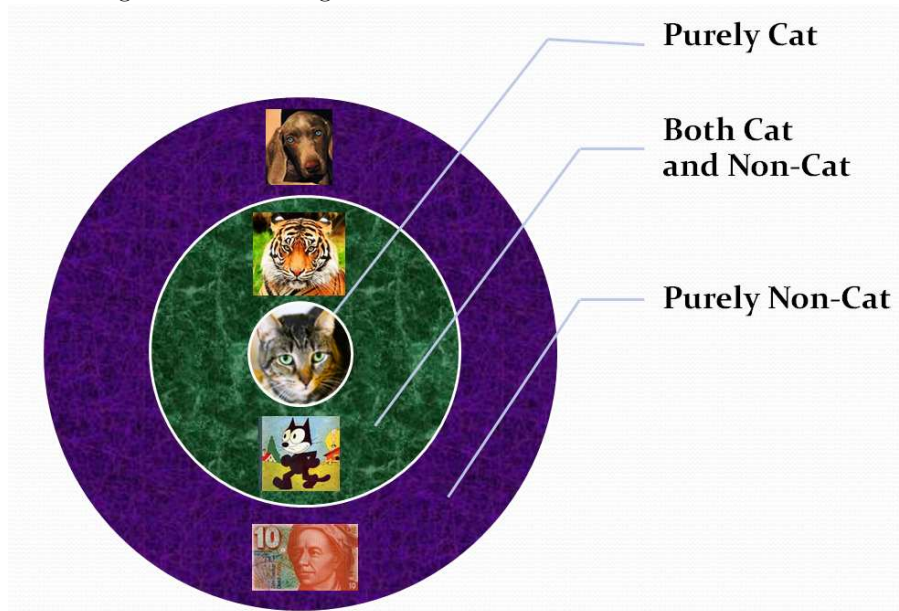
THOMAS DE KONINCK IS AND IS NOT THE LITTLE PRINCE

The same can be said in relation with Candy's favorite painting: *Lisa del Giocondo* is and is not *Mona Lisa*. Staying at the level of paintings, a fascinating topic whose philosophical dimension has not yet been fully explored (see [44]), we have the one by Magritte representing a pipe. This painting is popularly known in Las Vegas, Dubai and Shanghai as *Magritte's Pipe* but its original title in Smurf is *La Trahison des Images*. In English this is literarily translated by: *The Treachery of Images*.



A GIRL SMOKING A PIPE WHICH IS NOT A PIPE

Magritte said the following: “The famous pipe. How people reproached me for it! And yet, could you stuff my pipe? No, it’s just a representation, is it not? So if I had written on my picture *This is a pipe* I’d have been lying” ([50], p.71). But this would have been a true lie, because even if the *representation* of something is not the thing, whether it is an image, a word, an idea, nevertheless it shares something with the thing, deeply or superficially, the reason to say it also is the thing in some sense, at least nominalistically, for those who are afraid of ontologies or other things down to earth.



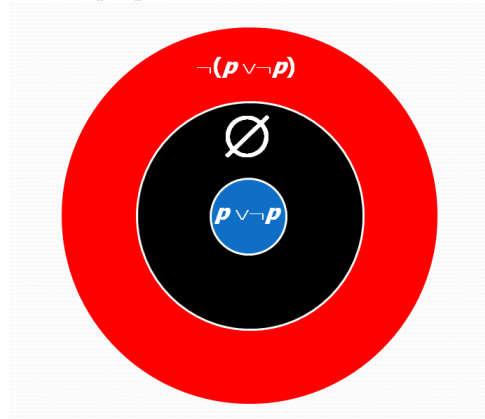
CIRCLING PARACONSISTENT NEGATION

Representation can be seen as the Queen of Paraconsistency, but there are also some princesses... Use if one of them. A chair is a chair, but a table, a stone or even a horse can be seen as chairs, if we use them as such. The stone on which the famous thinker is seated, the cornerstone of philosophy, is therefore a chair and not a chair. Some things are essentially chairs, some others only accidentally, according to Aristotle Rodin ...



THINKING ON A CHAIR WHICH IS NOT A CHAIR

We will let Candy look for other princesses. Right now we ask her: are there concepts without inconsistent zone? Properly consistent concept? Yes: to be or not to be a cat! We can say that about tautologies, propositions which are always true. *Felix is a cat* and *Felix is not a cat* can both be true, but they cannot be false together. *Felix is a cat or not a cat* is always true and therefore its negation is false. This proposition does not fall into the inconsistent zone.



CIRCLING A TAUTOLOGY WITH A PARACONSISTENT NEGATION

## 4 Happy End: Candy with her three Non-Cats

So at the end of this sunny day we have three non-cats:

- Classical Non-Cat
- Paracomplete Non-Cat
- Paraconsistent Non Cat



THREE NON-CATS IN CANDY'S HOUSE

How to deal with them? For sure they all need good food. But maybe they don't all need the same food to survive. Perhaps fish is better for the first, chicken for the second, lamb for the third ... They are not indeed built in the same way. And for each of these three species, there is not only one way to define or conceive it, even for the most straightforward of them, the classical one. We will not enter in technical details here, but we want to stress two important points:

- For the natural perspectives of both paracompleteness and paraconsistency presented here, there is the idea that paraphenomenology applies to everything except respectively antilogy and tautology.
- We have a duality between paracompleteness and paraconsistency.

The technology of logic systems for these two non-classical non-cats will be developed in a further paper, giving continuity to on-going works: either using possible world semantics in the line of the logic  $Z$  [9], either using the theory of valuations and sequent calculus in the line of the methodology which has been used for da Costa's systems [13], or three-valued truth-functional matrix semantics ([3], [21], [?]).

Note also that we have separately considered here paraconsistency and paracompleteness, but it is also possible to mix them, this leads to *paranormality* [46]. We will not present here a paranormal non-cat because it is better to go slowly. Candy may be terrified by such an animal. Let us first her tame the two other non-classical species of non-cats. We don't want our story to shortly end into a tragedy.



## 5 Souvenirs and Dedication

I don't remember exactly on the one hand when and how I first got in touch with John Woods, on the other hand when and where I met him in person for the first time, somewhere on this planet or on Boole's crater on the Moon? Or was it in the World of  $\bar{A}$ ? Anyway, I will point out here six important moments of our relationship:

- 2010 Vancouver
- 2012 Truth-values
- 2014 Vatican
- 2015 Istanbul
- 2016 Paraconsistent Newsletter
- 2018 Canadian Logic Prize

I met John in Vancouver in 2010 at the time I was doing an intellectual tourism trip with my wife Catherine crossing Canada from West to East. I gave a series of lectures in Vancouver (UBC and SFU), Calgary, Montréal and Québec City (where we had the pleasure to meet and conduct an interview with the little prince which is not the little prince, Thomas De Koninck, presented in the section 3 of this paper).

My talk at the Department of Philosophy of the University of British Columbia was on March 10, 2010 and the title was *Logic, logic and logics*, explaining the difference between logic as reasoning ("Logic") and logic as the theory of reasoning ("logic") (cf. [11]) as well as talking about the multiplicity of systems of logic that can be unified through a general theory ("Universal logic") which itself is not *a logic* (cf. [6]).

At this time I was working on a paper about truth-values for the impressive multi-volume *Handbook of the History of Logic* John was editing with Dov Gabbay (and additional editors for some volumes). In Vancouver John kindly gave me two volumes of the collection.

My paper entitled "History of truth-values" was included in the 11th and last volume of the series with subject *Logic: a history of its central concepts*, published in 2012. This paper is connected with the present one since I am talking about the general theory of valuations developed by Newton da Costa, which allows to deal with paraconsistent, paracomplete and paranormal negations. It is a lengthy paper of 72 pages which took me a lot of time to write.

I was glad that John liked it very much. Here his comments in a letter of support he wrote for me at some point: "His numerous essays on the history of logic are models of intellectual history at its best. A case in point is his *History of truth-values*, which I regard as indispensable reading for all logicians (especially the many teachers of logic who aren't)."

May 5-9, 2014 I organized the 4th World Congress on the Square of Opposition at the Pontifical Lateran University in the Vatican (cf. [28], [35]). SQUARE is a series of interdisciplinary events around the square of opposition, a logical mascot directly related with the topic of this paper. John was one of the keynote speakers of SQUARE'2014 presenting the lecture "How Globalization Makes Inconsistency Unrecognizable".

As I recall in my paper "The new rising of the square of opposition" [15], my interest for the square started when writing a review of Slater's paper criticizing the misuse of the word "contradiction" in paraconsistent logic (see [22]).

June 20-30, 2015 I organized the 5th World Congress and School on Universal Logic (UNILOG) in Istanbul, Turkey. UNILOG is a series of events promoting logic in all its aspects and also its relations with other topics: music, theory of colors, politics, physics, information, etc. John gave there a tutorial at the school entitled "Logic and Fiction".

**Paraconsistent Newsletter Summer 2016**

**THREE QUESTIONS TO JOHN HAYDEN WOODS**

1) When and how did you first hear about paraconsistent logic?

*In the fall of 1960 I read Anderson and Belnap's "A simple treatment of truth functions", JSL, 24 (1959), 301-312. In a footnote, it is mentioned without further comment that Disjunctive Syllogism fails in this treatment. "Well", I thought, "there goes negation or disjunction!" Later I read Belnap's technical report for the US Navy, A Formal Analysis of Entailment. "There goes ex falso", I said to myself. I learned from Ray Jennings in 1977 the name "paraconsistent" in application to logics of that sort.*



**John Woods - President of the University of Lethbridge 1979**

3) How do you see the evolution of paraconsistent logic? What are the future challenges?

*The paraconsistent programme has been overtaken by two philosophically important developments. One is the quickening pace of pluralism. The other is the tendency to favour technical virtuosity over philosophical elucidation. Each, in turn and in combination, place realist assumptions in serious jeopardy, in consequence of which logic has attained a diminished presence in the intellectual core of philosophy's programmes. "It's all just math, and none of it is true", said one of my students recently. To which he added, "Good!" Shortly after, he transferred to the mathematics department. If paraconsistent logic is to enjoy a philosophical*

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I conducted an interview with John for the Summer 2016 edition of the *Paraconsistent Newsletter*. I started this newsletter when I was working in Switzerland at the beginning of the 21th century. At this time I was just circulating this newsletter via e-mail. Then in 2016 I decided to create a new format, a webpage with pictures and music. For each edition (4 per year) there is an interview of someone related to Paraconsistency. There are 3 questions, always the same:

- When and how did you first hear about paraconsistent logic?
- How did you develop your work on paraconsistent logic?
- How do you see the evolution of paraconsistent logic? What are the future challenges?

John was kind enough to give proper answers to these three questions [58].

June 16-26, 2018 I organized in Vichy, France, the 6th UNILOG. At this event there was a *Logic Prizes Contest* gathering the winners of 9 logic prizes I managed to create, starting with the Newton da Costa logic prize for Brazil (for details, see [27]).

I invited John together with François Lepage (Université de Montréal) to be the organizers of the Canadian Logic Prize. John was quite enthusiastic about it and chose the name for it: *Schotch-Jennings logic prize*, with the names of Peter Schotch and Ray Jennings, two Canadian logicians whose work is related with paraconsistent logic.

Papers for logic prizes can be on any topic, but the fact that we have three prizes<sup>7</sup> with names of logicians connected to paraconsistent logic is not a mere coincidence. Paraconsistent logic which deals with the principle of non-contradiction, a central principle of thought and/or reality, is naturally at the heart of logic.

The organization of these prizes is not that easy, in particular funding is needed to send the winners to the contest at UNILOG. This is not always obvious to find a solution especially in a country like Canada where there is not yet a Canadian Logic Society. For this first edition John funded the prize (together with François) with his own pocket money. This shows his generosity and passion for logic.

For this Tributes Volume I decided to write a paper on the philosophy of paraconsistency, because John has been interested in the topic since many years (see in particular [57]). I started to seriously think about the ideas of this paper in 2009 when I was in Fortaleza, Northeast of Brazil. Then I presented them in many lectures, the latest and more directly related with the present paper was given in Dubrovnik at the congress FORMAL METHODS AND SCIENCE IN PHILOSOPHY II, May 4-6, 2017, where I presented a talk with the same title as this paper, *Cats that are not cats*.<sup>8</sup> This paper on paraconsistency is also connected to another favorite subject of John Woods, fiction (see [56]). I guess he will be happy to see Felix the Cat here and I hope he will enjoy the romanesque, not to say romantic, style of this paper, that I am very glad to dedicate to him.

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<sup>7</sup>The first is *Newton da Costa prize* for Brazil, the second *Schotch-Jennings logic prize* for Canada and the third *Vasiliev logic prize* for Russia.

<sup>8</sup>Thanks to Srećko Kovač and Kordula Świątorzecka for inviting me as a keynote speaker of this event.

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