Ex Incompatibilitate Sequitur Quodlibet (The Explosiveness of Incompatibility and the Compatibility of Negation)

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In memory of John Corcoran

Abstract. In this paper we explain why *Ex contradictione sequitur quodlibet* is a confusing expression to denote the statement $p, \neg p \vdash q$ and we also explain why this statement is ambiguous. We start by setting out a framework about consequence relation and truth. We proceed by presenting the basic concepts of the theory of opposition and the meaning of contradiction according to this theory. We then introduce the notion of incompatibility and, on the basis of that, we deal with explosion and introduce the notion of compatible negation. The final part of the paper is about John Corcoran.

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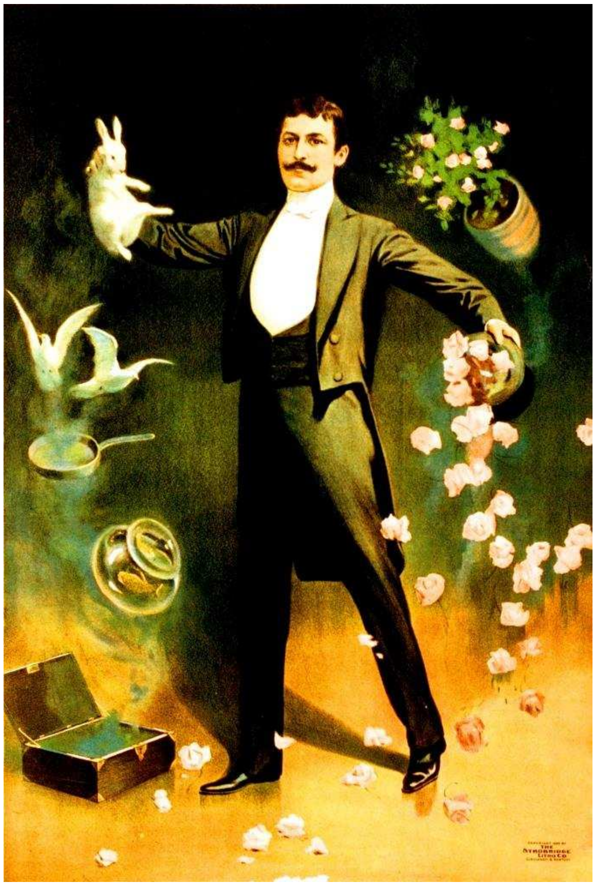
Keywords. contradiction, contrariety, compatibility, negation, truth, square of opposition, consequence, explosion, paraconsistent logic, John Corcoran.



Il y en a qui s'amusent à ébahir les péquenauds,
Avec des zéros qui ne tournent pas rond,
Des contradictions qui n'explosent pas,
Des paradis artificiels à prix d'enfer.
Baron de Chambourcy

1. The confusing *Ex contradictione sequitur quodlibet*

The main aim of this paper is to explain that it is confusing to call *Ex contradictione sequitur quodlibet* the following statement:

- (EX¬QL) $p, \neg p \vdash q$

It is a fact that if "¬" denotes classical or intuitionistic negation, EX¬QL holds, i.e. that from a proposition and its negation, follows any proposition.[1] The translation of *Ex contradictione sequitur quodlibet* in English is something like *From a contradiction anything you like follows.* An important point is to know whether $\{p, \neg p\}$ is a contradiction or not [21]. Classical negation can safely be called a *contradictory connective*, understood as an abbreviation for *contradictory forming connective*, considering that the pair $\{p, \neg p\}$ is always a contradiction according to the standard definition of *contradiction.* But to call intuitionistc negation a *contradictory connective* is highly ambiguous considering that intuitionisitc negation does not obey the principle of excluded middle. Therefore it is better to use another word than "contradictio" in this statement. We will propose one in this paper which is not ambiguous.

Another important point is the fact that one may disagree that a negation should obey EX¬QL. For this reason it is better not to use the symbol "¬" in the above statement and also not to use the symbolic acronym "EX¬QL". In logic there is no symbol and no logical constant for contradiction. If we represent contradiction by the symbol $\bigotimes$, we can symbolically write the *Ex contradictione sequitur quodlibet* as follows:

- (EX$\bigotimes$QL) $\bigotimes \vdash q$

There are two questions:

- Is it true that from a contradiction everything follows?
- Is there something else from which everything follows?

Beside the expression *Ex contradictione sequitur quodlibet*, there is also the expression *Ex falso sequitur quodlibet*, often considered as synonymous, which symbolically can be expressed as follows:[2]

- (EX⊥QL) $\bot \vdash q$

The symbol "⊥" is the logic constant called "Falsum", something which is always false, by contrast to the logic constant "⊤" called "Verum", something which is always true. These two symbols are also used in lattice theory meaning bottom and top, and these names are used to generate the corresponding symbols in the typesetting system LaTeX for mathematics. The symbol "⊤" is used in lattice theory because "T" is the first letter of "Top" and "⊥" is used because it is the reverse symbol and the bottom is conceived as the reverse of the top. In logic,

[1] We are using here the word "proposition", rather than "sentence" or "formula", not because we are dealing with a system of "propositional logic", but because we are talking about reasoning in general, a proposition being considered, in the spirit of Frege, a thought that can be true or false, see [48], [52] and [40].

[2] About the history of these expressions, see [50].

"T" is not the first letter of "Verum", but of "Truth". Note however that "Truth" in logic and its first letter "T" are also used to denote the truth-value true which is not the same thing as the constant denoting something which is always true.[3]

Something which is always true was called by Wittgenstein a "Tautology" ([67], 4.46). Wittgenstein did not invent the word "Tautology", but this word became famous with the meaning he gave to. A similar thing took place with "truth-value" and Frege [12]. "Tautology" also starts with a "T". Wittgenstein called something which is always false a "contradiction" ([67], 4.46), but later on he changed his views on contradiction. Anyway, a better name for something which is always false is "Antilogy".[4] The symbol "$\perp$" can be understood as corresponding to the notion of antilogy, and we can say *Ex antilogia sequitur quodlibet*, rather than *Ex falso sequitur quodlibet* avoiding the confusion between "Falsum" and the truth-value false. To avoid ambiguity we can use the acronym EXAQL instead of EX$\perp$QL. The important point is that EXAQL is indeed not equivalent to EX$\neg$QL unless we are dealing with some specific negation, like classical negation.

It is important to give the right name to the right thing. Many confusions (but not all) are rooted in language. An important task of philosophy is to improve the way we are using language. The objective of philosophy can be considered as establishing right distinctions and clarifying concepts. Language plays a fundamental role for doing that. From this perspective, philosophy is putting order in the room, rather than building castles in the sand (Fig 1). But putting order does not mean evacuating. Emptiness is the degree zero of order, it can however also be interpreted as the degree zero of disorder. Opposites meet in nothingness. And putting order also does not reduce to squaring everything, but to promote beautiful and complex structures. Nature is in order but it is not a French Garden. Music can also give an intuition about an order, which is neither silence, nor cacophony.

Figure 1 - Order in the Room and Castle in the Sand

[3]The connection between lattice theory and logic has been established by Stone and Tarski. Stone showed that a complemented distributive lattice is a Boolean algebra [56] and Tarski showed that a Boolean algebra is the algebraic structure corresponding to the quotient of the classical propositional structure, the so-called Lindenbaum-Tarski's algebra [58].

[4]But this not the original Greek sense of the word, see [2].

2. Consequence and Truth

In the previous section, the symbol "$\vdash$" was used in EX$\neg$QL, EX$\bigotimes$QL, EX$\perp$QL and EXAQL to denote a *consequence relation*, conceived as a relation between sets of propositions (*theories*) and propositions. A consequence relation can be defined in different ways: proof-theoretically, model-theoretically, ... or in an abstract way considering axioms or no axioms for it (see [4], [8], [11]). An *abstract Tarskian consequence relation* is a relation obeying the three basic Tarskian axioms: reflexivity, monotonicity and transitivity (see [57] and [9]).

Alfred Tarski also formulated the standard model-theoretical notion of logical consequence as follows:

- $T \models p$ iff $mod(T) \subseteq mod(p)$.

where T is a theory and p a proposition. In this definition a model is a "thing" according to which a proposition is true or (exclusive) false. This definition was introduced by Tarski in 1936 [59], but using neither the symbol "$\models$", nor the word "model". This terminology and notation were introduced only in the 1950s by Tarski when developing *Model Theory* [60]. According to Wilfrid Hodges, this idea of consequence can be traced back to *Abū alBarakāt* in the 12th century in Baghdad, see [51].[5]

It is easy to check that a consequence relation defined in this way obeys the three Tarskian axioms and that therefore it is an abstract Tarskian consequence relation.[6] On the other hand. an abstract Tarskian consequence relation can be defined model-theoretically by considering as models characteristic functions of closed theories (a theory is *closed* when it includes all propositions which are consequences of it), generally excluding the trivial theory, i.e. the set of all propositions (see [46]). These models are bivaluations whose values 1 and 0 are respectively called "truth" and "falsity", and for a given bivaluation β, we say $\beta(a) = 1$ rather than β is a model of a.

Considering these facts, when we have a connective $\star$ such that:

- (EX$\star$QL) $p, \star p \vdash q$

where $\vdash$ is an abstract consequence relation obeying the three Tarskian axioms, it means that p and $\star p$ don't have any common model, cannot be true together.

On the basis of that we can ask:

- Is it right to call the pair $\{p, \star p\}$ a *contradiction* ?
- Is it right to say that "$\star$" obeys the *Ex contradictione sequitur quodlibet* ?

Fortunately we can use the language as we want. The meaning of the words is not fixed for ever and is always changing. This is related to the evolution of thought: "one" does not mean the same now as before, for example at the time negative integers and zero were not conceived. A this time, one was the first! (see [24]). On the other hand, we must be careful playing with language, escaping

[5] About Tarski's two ways of defining the notion of consequence see [22] and [20].
[6] Tarski also did not use the symbol "$\vdash$", neither in its original meaning when introduced by Frege, nor in its abstract recent use [23].

sophisms. Someone may claim that the product of any number by one leads to one using the name “one” to denote zero, or that he has reached paradise calling Monaco the “paradise”. When using a word we must be careful about its common meaning and also how it has been defined or/and understood according to some important theories.

3. Contradiction according to the Theory of Opposition

We will present now the notion of contradiction from the point of view of the theory of the square of opposition. This theory is an important theory going back to Aristotle and which has not been seriously challenged by modern logic. In his famous booklet *Begriffsschrift* Frege presents a square of opposition, to support his theory of quantification [49]. There have been many recent developments of this theory, showing it is a very lively theory, with lots of applications (see [14], [27], [30], [31], [28], [29], [33] [34]).

We wrote “the notion of contradiction” because what we are talking about does not reduce to the word “contradiction” given to it. In fact this was not the original word used to talk about this notion, in particular due to the fact that the theory was first developed using Greek language. Also we wrote “the theory of the square of opposition” and not the “the square of opposition” because this theory did not start with a square and does not reduce to a square, although the square of opposition can be considered as an emblematic figure of this theory.

According to the theory of the square of opposition there are three notions of opposition, defined in the following way:

- Two propositions are *contradictory* iff they cannot be true together and they cannot be false together.
- Two propositions are *contrary* iff they cannot be true together but can be false together.
- Two propositions are *subcontrary* iff they can be true together but cannot be false together.

These three notions of opposition can be articulated in the figure of a square of opposition (Fig 2).

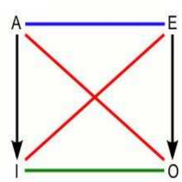


Figure 2 - The Square of Opposition

In this figure, blue represents contrariety, green subcontrariety, red contradiction and black subalternation, which is an implication. We have introduced this coloring, which has become standard, in [7]. “A”, “E”, “I”, “O” are the traditional names for the four corners. These corners were first conceived as propositions, but the theory also applies to concepts taken extensionally or intensionally.

If we consider in the framework of a Tarskian consequence relation two propositions k and t which are contradictory, we have:

- $k, t \vdash q$

for any proposition q. But this is also the case if k and t are contrary. Contradictory and contrary both share this consequence property. How to call this property? *Ex contradictione sequitur quodlibet* and *Ex contrario sequitur quodlibet* make both sense according respectively to the notions of contradiction and contrariety of the theory of opposition. But how to put them together, since they are exclusive?

4. Welcome to Incompatibility

Contradictory and contrary are themselves contrary notions, in the sense that something cannot be at the same time contradictory and contrary, but can be neither contradictory nor contrary. An interesting aspect of the theory of opposition is that it can be applied to itself: we have a triangle of contrariety about oppositions (Fig 3).

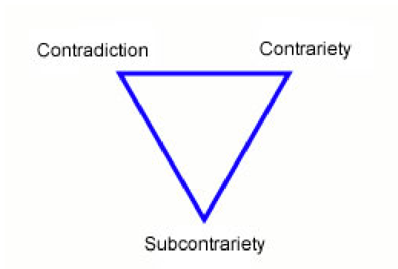


Figure 3 - The Contrary Triangle about Oppositions

From any triangle of contrariety, following Blanché's idea (see [35], [13]), we can construct a star and a hexagon (in which there are three squares of opposition) as shown in (Fig 4).

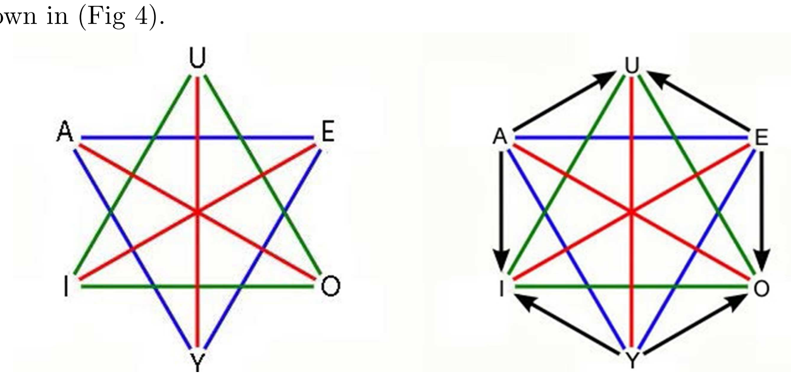


Figure 4 - Star and Hexagon of Opposition

Applying this construction to the contrary triangle of oppositions, we have a hexagon about oppositions (Fig 5).

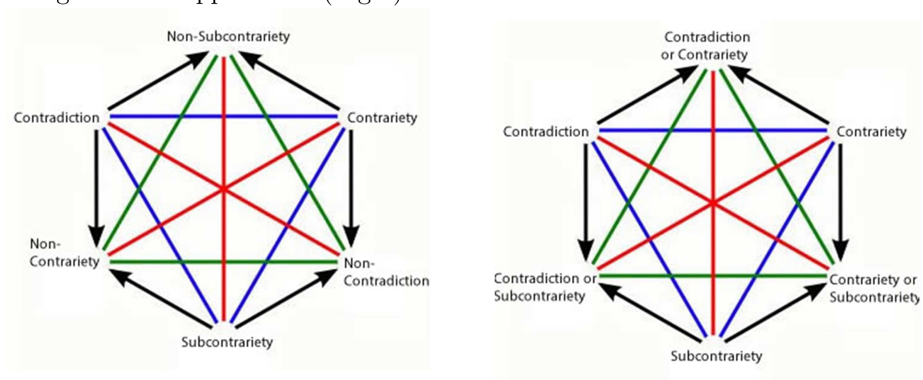


Figure 5 - Two Logical interpretations of the Oppositional Hexagon about Oppositions

These two interpretations are equivalent (the "or" is an exclusive or) and correspond to the logical structure of the hexagon. What is interesting is to find good primitive names for the three corners of the greeen triangle of subcontrariety, result of this construction. It is not always obvious, but in the present case for the top corner, which is the main object of interest for this paper, it is not so difficult, we can call it "incompatibility" (this terminology was introduced in this framework in [16]). It corresponds to "contrary or contradictory" and "non-subcontrariety" (Fig 6).

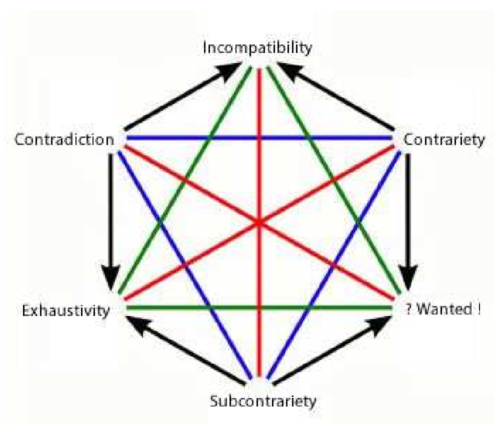


Figure 6 - The Hexagon of Incompatibility

The use of "incompatibility" in this context is in harmony with the usual sense of this word: "the state of not being able to exist or work with another person or thing because of basic differences" (Cambridge Dictionary). For use of this word in logic, linguistics and philosophy, see [3], [36], [38], [53], [54].

According to this definition, incompatibility does not mean that the two incompatible sides form necessarily a incomplete totality such as obligation and prohibition (optional is neither obligatory, nor prohibited), it also encompasses contradictory dichotomies such as identity and difference.

We have given the name "Hexagon of Incompatibility", because this is the notion we want to focus on. To give a name to a hexagon choosing the name of one of its corners is a common procedure (see e.g. [25]). Note that we also have given a name for the corner corresponding to "contradiction or subcontrariety", calling it "exhaustivity" (understood as an abbreviation of "exhaustive opposition").

Let us call a unary connective $\oplus$ an *incompatible connective* if the pair $\{p, \oplus p\}$ forms a contradiction or a contrariety. If a unary connective is incompatible, then (EX$\oplus$QL) holds and vice-versa.

- $p, \oplus p \vdash q$

For this reason we can call this statement *Ex Incompatibilitate sequitur quodlibet.*

5. Explosion, compatible negation and paraconsistent logic

The New Zealander logician Richard Routley (who moved to Australia and changed his name, becoming "Richard Sylvan") was acquainted with nuclear bombs and promoted the nickname *Principle of Explosion* to talk about the *Ex contradictione sequitur quodlibet.*

In this colorful context, it is worth emphasizing that not only contradiction is explosive, but also contrariety. Both are explosive! *Incompatibility is explosive.* "Explosion" is not necessarily a good name. For example pair and impair form a contradictory duo in the context of natural numbers and it seems that strictly speaking there is nothing explosive about these two contradictory notions, no Hiroshima effect, the same with the duo of domestic animals cat and dog (these two animals really form a contradictory pair in the context of domestic animals, unless we consider rats, cows or fleas as domestic animals) (Fig 7).

Figure 7 - Cat and Dog, a non-Explosive Contradictory Pair

In Chinese language the ideogram for contradiction is a shield/sword pair (Fig 8).

Figure 8 - Contradiction in Chinese

This is not so good because conflict / fight is just a very peculiar type of contradictions (see [32]).

The reason why Routley, and other deviant logicians, like this explosive terminology is because they think that the *Ex contradictione sequitur quodlibet* is dangerous. They defend the idea that contradiction is not explosive. Obviously by doing that they change the basic meaning of the word “contradiction”. This meaning change was strongly criticized by another pseudo-Australian logician. Barry Hartley Slater (Fig 9) (see [55], [10] and [17]):

> If we called what is now ’red’, ’blue’, and vice versa, would that show that pillar boxes are blue, and the sea is red? Surely the facts wouldn’t change, only the mode of expression of them. Likewise, if we called ’subcontraries’, ’contradictories’, would that show that ’it’s not red’ and ’it’s not blue’ were contradictories? Surely the same point holds. ([55], p.451)

To say that contradiction does not explode is indeed a bit like saying that 0 times any number does not yield to nullity. This is perhaps good for *épater les bourgeois*, or *ébahir les péquenauds*, but this is not philosophically very profound.

Figure 9 - JYB and Hartley Slater discussing
the future of paraconsistent logic at the 11th LMPS in Kraków in 1999

If someone wants to support the idea that a negation can be a non-explosive connective, this is the basis of *paraconsistent logic* (see [5]), it is not necessary to claim that contradiction is non-explosive. We can say that a *paraconsistent negation is not explosive* or, to speak in a more peaceful way, that a *paraconsistent negation is not an incompatibile connective*, that a proposition p and its negation $\neg p$ can be compatible. It means that p and $\neg p$ can be true together. And there is no necessity to speak about "true contradiction", since p and $\neg p$ in this case do not form a contradictory pair, but a compatible pair. Following this terminology, the basic idea of a paraconsistent negation is that *negation can be compatible*, i.e. negation can be a compatible connective. This terminology is good to clean paraconsistent from contradiction, or to use a liberal way of speaking, to free paraconsistency from contradiction (cf. [1]).

6. The existential import of John Corcoran

As far as I remember, I first met John Corcoran through his masterful introduction to the second edition of the collection of papers by Alfred Tarksi, *Logic, Semantic, Metamathematics*[62]. Here are two comments about this volume:

> "I can think of no better publishing project in the general area of logic and the foundations of mathematics than the republication and appropriate corrections under Tarski's supervision of this classic volume."
> Patrick Suppes, Stanford University
>
> "A mere reprinting of the volume would be a service, but the present project promises a much greater service. The historical significance of the papers can now be assessed in the perspective of the twenty-six productive years that have elapsed since the first printing of the volume,

> and the much longer intervalfifty years, on the average since publication of the component papers. Concepts can be instructively glossed, renamed, and reinterpreted in the light of later literature. Corrections can be made, also, that were urgently wanted already in the first printing, to which Tarski had insufficient access when it was being prepared. Under the expert editing by John Corcoran in consultation with Tarski, a volume can be counted on that will constitute a definitive record and appraisal of Tarski's monumental early contributions to the burgeoning domain of mathematical logic and its philosophy." W. V. Quine, Harvard University

This collection of papers is on a variety of topics and it is not obvious to make connection between all of them. Tarski was impressed that Corcoran was able to do so in this introduction. Corcoran was a good friend of Tarski. He attended his talk on logical notions on April 20, 1973, at the State University of New York at Buffalo, USA and posthumously published the seminal paper corresponding to it [63].[7]

I read his introduction to Tarski's volume when I was a student of logic in Paris at the end of the 1980s. In 1995 I was a Fulbright research scholar at the department of mathematics of UCLA (University of California, Los Angeles), invited by Herbert Enderton. I used also to go to some classes and seminars at the philosophy department (as were going Yiannis Moschovakis and Tony Martin). One day Joseph Almog in his philosophy class brought a paper he had recently discovered that he thought was very good, by a completely unknown author, he said. This was the paper "Categoricity" by John Corcoran published in the first issue of the journal *History and Philosophy of Logic* [39]. That Corcoran was unknown in Los Angeles seemed to me a bit strange at this time. But later on I noticed than this fact was common in United States. People were part of small local communities with no relations not only with the rest of the world but also with the rest of United States.

This communitarianism is in fact something going on everywhere in the world and it is a limitation for the progress of science. I have tried to change this situation with the development of universal logic, which is at the same time a universal approach to logic and the promotion of interaction between all logicians around the world [19]. I organized the first UNILOG (World Congress and School on Universal Logic) in Montreux (Switzerland 2005), then in Xi'an (China 2007), Lisbon (Portugal 2010), Rio de Janeiro (Brazil 2013), Istanbul (Turkey 2015), Vichy (France 2018), Chania (Crete 2022). The next edition will be in Cusco (Peru 2025). I invited Corcoran to come to the 5th edition in Istanbul to give a tutorial on Aristotle. He enjoyed very much the meeting and the city (Fig 10).

[7] Tarski is with Kurt Gödel the most famous logicians of the modern area. Funny enough, Tarski was born the same day of the year as Gödel died, January 14. Based on this coincidence, I launched the World Logic Day on January 14, 2019 (see [18]) and I succeeded to have it recognized by UNESCO the same year.

Figure 10 - Corocoran lecturing at th 5th UNILOG in Istanbul in 2015

I had met Corcoran for the first time in person in 2008 in Paraty, Brazil, at the *15th the Brazilian Logic Meeting* to which my colleague Itala D'Ottaviano had invited him. But I was continuously in touch with him by email before and after that. I proposed him to come to the *2nd World Congress on the Square of Opposition* in Corsica in 2010, but he was not able to come, although much interested in the topic. He sent me later on an email about his paper "Existential import today: New metatheorems; historical, philosophical, and pedagogical misconceptions" [42], commenting: "Ranked first on the "Most-read list" at *History and Philosophy of Logic* with over 8000 readers, this demanding but self-contained and widely accessible paper grounded in standard first-order logic refutes over a century of mistakes about existential import."

John took part to many of the projects I have been involved in:

- He kindly wrote a paper (with Idris Samawi Hamid) for the two-volume book for my 50th Birthday "Investigating Knowledge and Opinion" [41].
- He contributed to the first issue of the *South American Journal of Logic* I launched with my colleague Marcelo Coniglio in 2015 with the paper "Tarski's Convention T: Condition Beta" (with Leonardo Weber) [43].
- He supported my project to create a book series with College Publications to publish the most important PhDs in logic. The first volume of the series (2017) was the PhD of Haskell Curry originally written in German *Grundlagen der kombinatorischen Logik* (PhD defended in Göttingen under David Hilbert) [47].
- He contributed to a special issue of *Logica Universalis* for UNILOG'2018 in Vichy France, with the paper "Aristotle's Prototype Rule-Based Underlying Logic" [44].

I started to discuss with John a project that I hope to conclude in the next future. Tarski's collected papers were published by Birkhäuser in 1986, after his death [64]. During many years, these volumes were out of print. I told Birkhäuser that it would be important to reprint them. It took a while because there were some technico-adminstrative questions to solve, but finally this was done in 2019. At the same time I proposed to prepare (with John) a fifth volume with additional materials not included in these volumes, such as the posthumous paper edited by John on logical notions [63], another posthumous paper published by Wolenski also in HPL [66], the famous piece published in *Scientific American*, “Truth and Proof” [61], some letters such as Tarski’s letters to Neurath explaining in particular his relation with Gödel and the incompleteness theorem [65].

Anyway, my relation with John Corcoran had a conclusive and happy end. In 2020 I was asked by Andrew Schumann to write a paper in honor of the 80th birthday of Jan Wolenski. I decided to write a paper on logical notions, in particular applying the square of opposition to logical notions. John carefully read several versions of this paper and helped me to seriously improve it. This email exchange took place shortly before his death, and here is the result: ”The Mystery of the Fifth Logical Notion (Alice in the Wonderful Land of Logical Notions)” [26].

The reason why I decided to dedicate the present paper about the *quodilbet* him is twofold. On the one hand, the topic is related to one of his main interests, Aristotle’s theory of opposition. On the other hand, the aim of this piece is to elucidate some basic notions of logic, with clarity and precision, in the line of the methodology promoted by John Corcoran.

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