Identification of Identity

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## 1 Identity: between contradiction and tautology

Wittgenstein wrote in the *Tractatus* (5.5303): "to say of *two* things that they are identical is nonsense, and to say of *one* thing that it is identical with itself is to say nothing at all". From this point of view, we can say that identity is swinging between the Charybdis of contradiction and the Scylla of tautology. Identity is a fundamental process in thought. Though is moving in a subtle way and it is important to understand how it works, how it finds its way between Charybdis and Scylla.

When we are thinking, we are identifying things which are different. Identity and difference go together. A *concept*, like for example the cat concept, consists in identifying a series of different objects. Among animals, we are unifying a class of them who have the same features or properties. At the same time, we know that each cat is different from the other ones. We are making abstraction of these differences. For *individuality* there is also an identity process. When we are talking about the Amazon river, we are identifying by abstraction a series of phenomena out of time and space. To understand how these two identification processes work — conceptualization and individualization — is to understand the Charybdis of identity.

The Scylla of identity can be examined through what we can call trivial identity which is described by the following version of the principle of identity: *everything is identical to itself and different from other things.* Sometimes the principle of identity is expressed only as the first half of this sentence: *everything is identical to itself.* Surely this is not enough to characterize identity since any reflexive relation obeys this axiom. Identity has to be reflexive but *only* reflexive. The second part of the principle of identity does not help to solve the problem, because *different from other things*, means different from different things, or not identical to non identical things, it is a tautology. At the end *everything is identical to itself and different from other things* is equivalent to *everything is identical to itself*, that is to say just reflexivity, which is not enough.

In a sense, trivial identity is inexpressible in ordinary language and it is also non axiomatizable in first-order logic (see e.g. Hodges 1983), but at the same time it can be expressed through mathematical diagrams: the diagonal of the Cartesian product of a set or with circular arrows around each point of a graph. The Scylla of identity can therefore be visualized. It is quite nothing, but it is something.
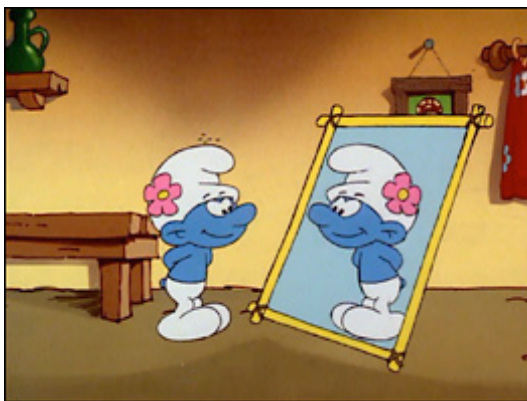
2 Naming identification and naming differentiation for singular terms

Quine wrote about the above quote of Wittgenstein that the logico-philosopher was mistaken and that "Actually of course the statements of identity that are true and not idle consist of unlike singular terms that refer to the same thing" (*Word and Object*, p.117). So the question of reference would be a way to navigate through the Scylla and the Charybdis of identity. But Quine is touching here only one aspect of the problem, there are many other ways to tame the two monsters. The same

thing can be named by different names, but also different things can be named by the same name, and these two processes do not concern only singular terms, but also plural terms.

Brasilia and *the Capital of Brazil* can be considered as two different names for the same thing: the city of Brasilia. *Rio de Janeiro* can be considered as the same name for two different things: the city of Rio de Janeiro as it is now and the city as it was before. We can call the first process *naming differentiation* and the second one, *naming identification.*

Now we have to understand why we are doing so. This use of language certainly reflects some fundamental processes of thought that have to be clarified. Frege wanted to avoid this use of language, to develop a perfect language where there will be no such ambiguities (Frege 1883), and Wittgenstein was following the same line when writing: "Identity of the object I express by identity of the sign and not by means of a sign of identity. Difference of the objects by difference of the signs." (*Tractatus*, 5.53).

But in fact these ambiguities are part of the very nature of thought, we donÕt have to get rid of them — it is too easy, a kind of bulldozer methodology — they have to be properly understood. This is in some sense what people have tried to do in the stream of philosophy of language, using in particular the distinctions between sense and reference, definite descriptions and proper names, and the notion of possible world. This may give interesting hints but does not explain everything.

The Capital of Brazil was a name for *Rio de Janeiro* in the state of the world in 1950 and is now in the present world a name for *Brasilia*. We are performing a

naming identification, using the same name for two different things. This is based on a clear relation between two possible worlds, the time line, and on the fact that to be a capital of a country is a precise feature that does not characterize a city and may vary through time.

The Capital of Brazil is considered as a definite description and one may think that it is its very nature of definite description that explains the naming identification. By opposition, some people, in particular Kripke, have argued that a proper name like *Rio de Janeiro* would always refer to the same thing in all variations of the world, in all possible worlds. But surely *Rio de Janeiro*, as it is now, is not exactly as it was before: there are more people, more pollution and it is not anymore the capital of Brazil. But some people would say: Rio de Janeiro will always be Rio de Janeiro, as if there was something permanent, essential which characterizes the town, a kind of spirit of the city. But maybe it is the use of language itself, the use of a proper name that solidifies the variations. This proper name means nothing, then for this very reason its meaning does not vary, such kind of name is a fix point to something fuzzy.

The difference between proper names and definite descriptions is that in the case of proper names, it constitutes the basis of identity (two objects are considered as essentially the same). It is a *proper naming identification*. In case of definite descriptions it is just a feature in common (two recognizable different objects have a common feature), which can be essential or not. It is a *descriptive naming identification*. Definite descriptions are based on a more rational identification and proper names on a more creative one, not to say artificial one.

Let us now turn to naming differentiation. *The capital of Brazil* and *Brasilia* are two names for one thing in the present world, one name is a definite description, the

other one is a proper name. *The capital of Brazil* is a name which refers to a city in function of a certain feature, this feature may be accidental. Why referring to a city in an accidental way? Because in some context this feature is the most important, it gives the adequate information, for example when we are saying: *The president of France is traveling to the capital of Brazil.*

Name differentiations may be based on ignorance, like the story of the morning star and the evening star. I can call a mouse Mini and another mouse Kiki and at the end of the day see that there are one and the same mouse. It can also be used to create a difference when there is not necessarily one: Norma Jeane Mortenson was also called Marylyn Monroe. The splitting of one person through the use of two proper names may lead to suicide or schizophrenia.

3 Naming identification and naming differentiation for plural terms

These two processes can also happen for plural terms. Note first that the distinction between proper names and definite descriptions can also be made for plural terms: Inhabitants of Rio de Janeiro is a definite description and Carioca is a proper name to refer to the same people. So here we have an example of naming differentiation which works quite in the same way as for naming differentiation for singular terms.

To use a definite description in case of plural terms is a way to capture a collection of objects by one feature they have in common which can be more or less accidental, such as red objects, edible mushrooms or hot dogs. In this case we can speak about loose identity. But one may want to find a feature which captures an essential

feature such as when we say that *human beings* are *rational animals*. The idea of rational animal is a way to unify a group of animals at the same time characterizing them, as if the identity was real. The identification process is then stronger than to use a proper concept (i.e. a concept working like a proper name) which is just a name without particular meaning, like *human beings*. Using a proper concept may be quite ambiguous because it is a way to artificially build an identity made of fuzzy features through a tag, like in nationalism: I am proud of being a Poldavian.

Let us now consider a limit case, the concept of *object* in the wide sense of the word, something which exists or can be thought: a cat is an object, anger is an object, Donald Duck is an object. They all are objects, what they have in common is to be an object, all are the same because all are objects. This is the opposite of individuality through the principle of identity according to which each object is identical to itself but different from the other ones.

4 Identification and differentiation in mathematics

It is important to stress that although language may have an important role in the process of identity, identification and differentiation may be considered beyond language and names. Identity can be seen as a relation between objects and/or between thoughts. This is not necessarily obvious for people who put language in the first place and have the idea that it is predominant in the thought process. This is generally the case of philosophers of language, who are studying thought through some analyses of language, like the distinction between definite descriptions and proper names. And it is also the case of modern logicians who have studied reasoning through the construction of formal languages.

But mathematics can be considered as a good example where naming identification and naming differentiation can be understood beyond a pure naming process, where identification and differentiation are performed beyond language.

As an example of identification, we can examine the case of zero: zero as a natural number and *zero* as a rational number are different, they have different properties. The reason why the same name is used is that they have also some common features which can be expressed in particular by the fact that the structure of natural numbers is a substructure of the structure of rational numbers. The notion of structure, which is the key notion of modern mathematics, permits to explain how identity works.

Some people may have the idea that a number is an entity with some mysterious features hidden behind a symbol, a proper name, like 7. But according to modern mathematics an object is nothing else that its relations with other objects in a

structure, which also can be understood by its relations with other structures. To identify an object is to identify a structure, or better, a class of structures.

The identity of zero is its properties in a given structure or/and in a class of structures. Several solutions are possible. As we were saying we may want to identify the zero of the natural numbers with the zero of the rational numbers, but their identity here is based on algebraic characteristics with regards to multiplication and addition. Now from the point of view of order, zero is quite different as a natural number and as a rational number. It makes no sense to say that there is one true zero. The zero of the natural number maybe is the firstborn but it has grown and has many aspects: the zero of rational numbers, the zero of real numbers or something like the empty set — then another name is used, maybe not because its properties are so much different but rather because it is conceived from a different point of view. This could appear as a naming differentiation.
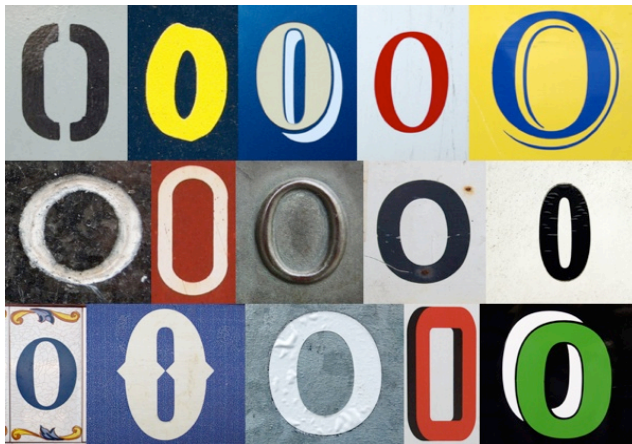
One may want to characterize precisely the zero of natural numbers. This can be done for example through Peano axioms of arithmetic, in first-order logic or in second order logic. As it is known, in first-order logic the theory is not categorical, there are several non-isomorphic models and zero is not the same in these different models. It is not the same for example in the standard model and in a non-standard model where there are other numbers after the standard natural numbers that are out of reach of zero through the successor function. In first-order logic, we are able to characterize zero with some axioms, however these axioms characterize not just one zero, but a multiplicity of zeros that are identified through the notions of axioms and models. Model theory is a way to understand the identification process. In second-order logic it is possible to have a categorical axiomatization of arithmetic,

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where all models are isomorphic (given a fixed cardinality). In this case zero is always the same up to isomorphism.

Isomorphism is identity between structures. It is the easiest way to identify structures. Now there is a more complex one which is equivalence between structures in the following sense: two structures are equivalent if they have a common expansion by definition up to isomorphism. An example of such equivalence is between an idempotent ring and a complemented distributive lattice. Equivalence between structures can be compared with translations between languages, which are not therefore not generally considered just as simple isomorphisms. If one language can be translated into another language and vice versa we can say that they are identical. In this sense the Bible in English can be considered as identical to the Bible in French.

Now let us see how (naming) differentiation works in mathematics. The differentiation process can be clearly explained using the concept of congruence: two objects are different but can be identified through congruence. For example *p&p* is congruent to p, according to the notion of logical equivalence which is a congruence in classical propositional logic. We are using two different names, "*p&p*" and "*p*", to denote two different objects, that can be seen as the same. In the original structure they are different, in the factor structure, they are the same. Differentiation is based on identification.

In mathematics a structure where there are no non trivial congruences is called a simple structure. In such a structure two different objects cannot be identified according to a congruence. For example the structure of natural numbers is a simple structure: it is possible to identify 2 and 4 saying that they are both even numbers, but they cannot be identified through a congruence relation, they are really different.

5   Leibniz principle of identity and congruencies

According to something which is generally called "Leibniz principle of identity", *two objects are identical if and only if they have the same properties*. This principle can be split in two: *identity of indistinguishable* (if two objects have the same properties they are the same) and *indistinguishability of identical* (If two objects are the same, they have the same properties).

Indistinguishability of identical is nothing else than congruence. In a given structure there is a whole hierarchy of congruence relations, the lattice of congruencies: Leibniz identity is the strongest one, the principle of identity as we were presenting it at the beginning (*everything object is identical to itself but different from other objects*) is the weakest one. If we have a simple structure, Leibniz identity is the

same as the principle of identity, this is the only congruence.
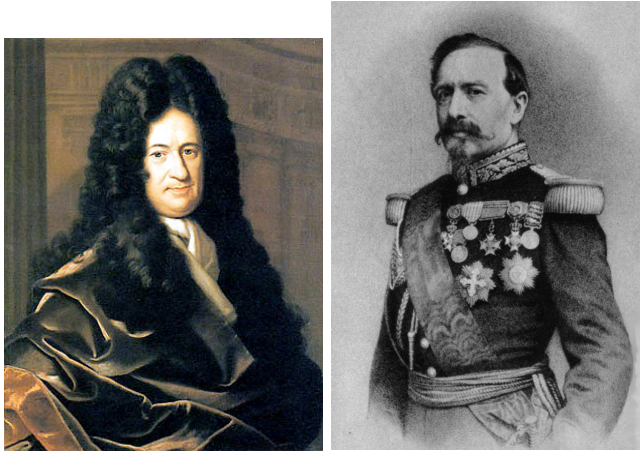
When a structure is not simple, *two different* objects can be the same in the sense of Leibniz. But one can say that the only difference is that they are two, not just one, we can speak about numerical difference in this case. An example is a structure with a two element set, Bibi and Titi, and with a universal binary relation: every object is related to every object. In this case Bibi and Titi are the same according to Leibniz identity. Leibniz identity is not so interesting, it is a trivial case like the identity given by the principle of identity. What is interesting is to identify things that are different in a sense which is not purely numerical.

This is what happens with non trivial congruences, which is probably the most precise formulation of what a subtle identity can be beyond Charybdis and Scylla. This notion is a central tool in modern mathematics and is directly connected with another key notion of modern mathematics: the notion of morphism - since a morphism generates a congruence and vice versa. The notion of congruence in its generality was presented by the General Bourbaki. Maybe only a Cretan warrior could find the way through Charybdis and Scylla.

6 Dedication and personal recollections

I had many interests in commons with Dale Jacquette: Boole, Schopenhauer, the Square of Opposition, ... And like me Dale was working on many different topics. He moved to Switzerland shortly after I left the country. I visited him in Bern when I was working in Brazil. I gave a talk there at the Institute of Philosophy and we had dinner together, with his wife Tina and my wife Catherine. I remember that he

told me that one of the reasons he left USA was because its department (at Penn Sate University) was dominated by continental philosophy. He took part to three editions of the series of congress I am organizing on the square of opposition: the 2nd edition in Corsica in 2010, the 3rd edition in Beirut in 2012 and the 4th edition in the Vatican in 2014. We published together a book resulting from the Corsica edition (Beziau-Jacquette 2012).

Dale Jacquette at the 3rd World Congress on the Square of Opposition American University of Beirut, Lebanon, June 2012.

He took also part to two editions of another series of events I am organizing, UNILOG: World Congress and School on Universal Logic. the 3rd UNILOG that took place in Lisbon in April 2010 and the 5th UNILOG organized in Istanbul in June 2015 where he gave a tutorial on Boole. The present paper deals with a topic Dale liked a lot: identity. His B.A honor thesis (1975) was on Aristotle on identity and identification. Along his carrier, this was a recurrent topic, see e.g. his 2011 paper "Frege on Identity as a Relation of Names". We posthumously published in the South American Journal of Logic his piece "Anatomy of a Nonidentity Paradox" (2016). I have myself been working on this topic since more than 20 years. This paper is the 10th in a series of paper I wrote on identity and I am glad to dedicate it to Dale.

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