Is logic exceptional ?

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Abstract We examine in which sense logic can be considered as exceptional. We start by emphasizing the difference between *Logic* as reasoning and *logic* as the science of reasoning, an essential distinction to launch the discussion. We then investigate if reasoning itself can be seen as exceptional, in particular an exceptional feature of human beings, and next if the science of reasoning can be regarded as exceptional. This study is further extended on the one hand by discussing the relativity and universality of logic, on the other hand by stressing the dialectical interaction between logic and metalogic, the interdisciplinary and transdisciplinary character of logic.

Keywords logic, metalogic, reasoning, universal logic, square of opposition, negation.

Mathematics Subject Classification Primary 03A05 Secondary 01A55; 01A60; 03-03; 03B53

1 Challenging the exceptionality of logic

Logic is still a mysterious and not so well-known topic. There are different reasons for that. For the layman, logic is first of all logical, know as an adjective not a substantive. He has not idea that there is a substance corresponding to logic, and that there is a substantial science of reasoning.

At the universities there rarely are some departments of logic, logic is spread in various departments and faculties where it is taught in different ways and there are few interactions between the different aspects of logic: mathematical, philosophical, computational, semiotical.

Despite the fact that during many centuries since Aristotle logic was a key to understanding and that very famous thinkers of the 20th of the century can without doubt be considered as logicians - Gödel, Russell, Wittgenstein, Turing - there is a tendency nowadays to dismiss the value of logic, considering that it is a stuff like any other one, nothing exceptional … (see e.g. [35]).

We take the opportunity of this post-modern analytic philosophy's view to emphasize how much logic is extraordinary.

2 Logic, logic and λogic

When talking about logic, it is important to make the distinction between *logic as reasoning* and *logic as the study of reasoning*. We have proposed [4] to graphically represent this distinction naming reasoning by "Logic" and the study of reasoning by "logic", by analogy with the distinction between "History" and "history", where we also have the use of the same word for the object of study and the study of this object, by contrast with e.g. *Linguistics*, the study of *language*, where we have two words with a common root, but which are nevertheless clearly distinct.

We can use the word "λogic" when talking both about logic as reasoning and logic as the study of reasoning (Fig.1). This double articulation is already a distinctive feature of λogic, one reason to consider it as exceptional. But this "reason" is shared with what we can similarly name "ηistory". We can put these two exceptionalities together talking about the "ηistory of λogic".

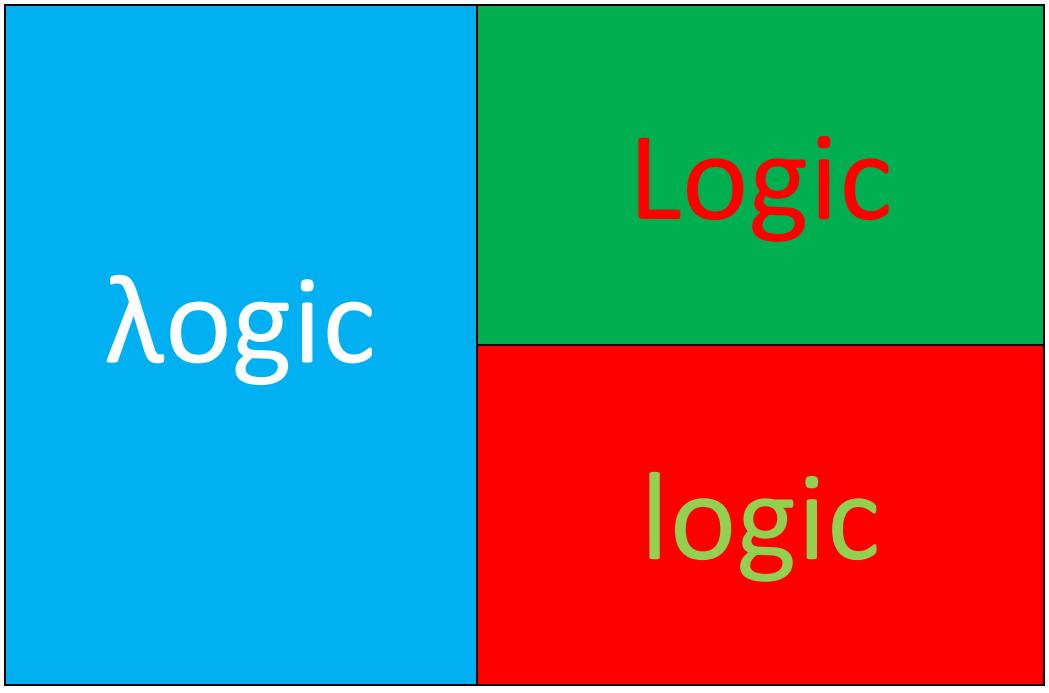

Fig. 1 The double aspect of logic

The reason to put reason in quotes is to emphasize that we are reasoning. We are reasoning about reasoning and the science of reasoning; we are reasoning about λogic. And this is the difference between λogic and ηistory, showing that λogic is more exceptional than ηistory.

The distinction between *logic* and *Logic* is important to answer fundamental questions about *λogic*, like the question *Is logic exceptional?* According to our distinction, we have here two questions: *Is reasoning exceptional?* and *Is the study of reasoning exceptional?* These two questions are different and so are their answers, although they are interrelated. We study them one after the other.

3. Is Logic exceptional?

Reasoning has been considered as a characteristic feature of human beings [11]. They were called for this "reason", *rational animals*. This terminology mixes together two different claims: human beings are the only animals who are reasoning, reasoning is the main or/and only distinctive feature of human beings with respect to animals. Let us draw the following table:

(RA)	Human beings are rational animals.
(RAO)	Human beings are the only animals who are reasoning.
(RAM)	Reasoning is the main or/and only distinctive feature of human beings with respect to animals.

(RA) can be understood as (RAO) and/or (RAM). (RAM) implies (RAO) but not the converse. So (RA) can be understood as (RAM), or (RAO) but not (RAM). At the end we have two possible interpretations of (RA) described in the table below.

(RAM)	Reasoning is the main or/and only distinctive feature of human beings with respect to animals.
(RAW)	Human beings are the only animals who are reasoning, but there are other important features distinguishing them from other animals.

(RAM) and (RAW) cannot both be true, but they can both be false, they are therefore contrary, according to the theory of opposition, most

famously represented by the square of opposition (see [20], [21], [22], [23]).

If we consider that (RAM) is true, it is a good reason to consider that *Logic* is exceptional. If we consider that (RAW) is true, we can consider that *Logic* is exceptional in a weaker sense. If we consider that both (RAM) and (RAW) are false, it is still possible to consider that *Logic* is exceptional if reasoning is viewed as an important feature of those *beings* who are reasoning, or an important *phenomenon* by itself.

We are using these expressions because *reasoning* can be considered as a feature not only of animals, but also of non-biological entities like computational devices (considered as physical entities, like your personal computer or theoretical entities, like a Turing machine), or divine entities (cf. John 1:1 identifying God with the Logos), or the universe/world itself (cf. *Nihil est sine ratione*).

To properly answer the question *Is Logic exceptional?* we have to examine the very nature, if any, of reasoning and all its possible manifestations. This can de done through the science of reasoning.

4. Is logic exceptional?

Let us examine now if the study of reasoning is exceptional. If we consider that reasoning itself is exceptional, we can consider that just for this reason the study of reasoning is exceptional, the study of an exceptional thing being exceptional. But one may want to relativize things even in this case, saying that the study of an exceptional thing can be carried out in a standard way, the study of dinosaurs being at the same level as the study of rats, although one may need a bigger laboratory to study dinosaurs.

On the other hand, if one considers that reasoning is not exceptional, this also does not rule out the possibility to consider the study of reasoning as exceptional. There are indeed many possibilities and to find our way out, we need to have a closer look at what the study of reasoning is.

In the study of reasoning, we can distinguish three levels (Fig. 2):

- System
- Theory
- Science

There are many *systems of logic*: syllogistic, classical propositional logic, the modal logic S5, to cite three famous ones. These systems are studied in different ways, using different frameworks: proof theory, model theory,

recursion theory, set theory. This is the *theoretical level*. The science of logic includes all this and more, in particular philosophical, historical and semiotic aspects.

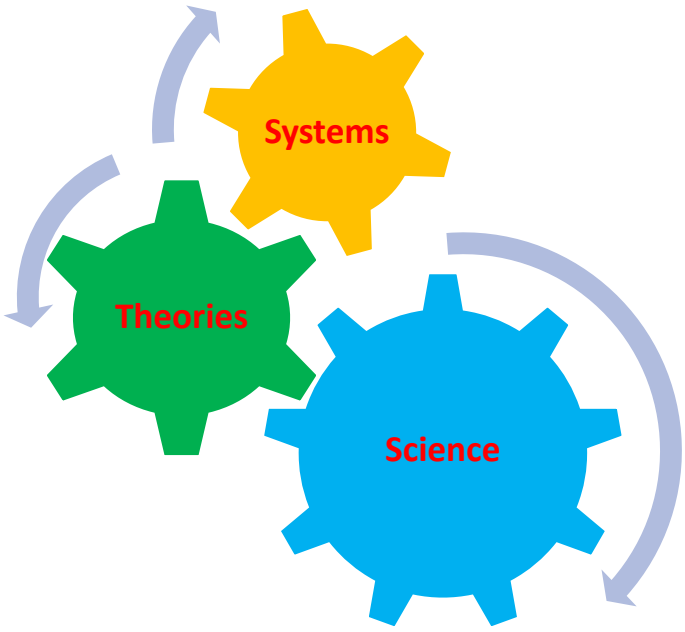

Fig. 2 Three levels of the study of logic

We can identify the *study of reasoning* with the *science of reasoning*, but it can be useful to distinguish the two and to have two different terminologies. Firstly, to accentuate the difference between the science of logic and a particular system of logic or a particular theory. Secondly to emphasize that the science of logic is not just all the studies of reasoning put together in a bag, or even a synthesis of them, but a dynamic articulation with additional features both as means and results.

These three levels and a similar process exist in any discipline. So, logic is not exceptional in this sense. One may sustain that logic is exceptional in:

(EA) The way these three levels are articulated.

(EP) The place of logic regarding other disciplines.

(ES) The relation between *logic* and its object of study.

And there is an additional fourth point that needs to be examined, it is the question of the development of logic, question which is related with the three points about.

(EE) Evolution of logic.

In the next sections we will examine these four points from different perspectives and show that logic is fourfoldlyy exceptional.

5. Is λogic relative ?

There are two important theories in modern science: the theory of evolution and the theory of relativity. From a scientific perspective these are two relatively precise theories evolving, about some determined fields: biology and the physical world. But there are some ideologies surrounding them which are shaping the way of thinking of human beings and also their behavior, in particular due to the ambiguous meanings of the words "evolution" and "relativity", words which are not innocent.

It seems that the theory of relativity is nowadays ruling the world. Everything is relative, even logic … This is an application of the rule of the universal quantifier: $\forall x R(x)$, therefore $R(logic)$.

Louis Rougier (1889-1982) made the following claim: "Avec la découverte du caractère conventionnel et relatif de la Logique, l'esprit humain a brûlé sa dernière idole" ("with the discovery of the conventional and relative character of Logic, the human spirit burned its last idol") [44] and published a paper entitled "The relativity of logic" in 1941 [43].

Once again, if we don't make the distinction between *logic* and *Logic*, the situation may be quite confusing and Rougier is mixing the two, attacking simultaneously the relativity of *logic* and *Logic.* He is against rationalism, arguing that general principles, such as *the whole is greater than the part*, on which it is based, are meaningless. And he is in favor of the multiplicity of systems of logic [6].

We can say that the science of logic, like any other science, is evolving, in the sense that it is changing, there are some new discoveries, so logic is not exceptional in this sense. This is was not necessarily clear before the rise of modern logic. Everybody knows the famous claim of Kant, who thought that logic was *a priori* and that Aristotle's logic perfectly describes the laws of reasoning, similarly as Newton's physics perfectly describes the law of material objects.

It is interesting to make the parallel between logic and physics: the fact that the physical science is evolving does not imply that the laws of the physical world are evolving. In the same way one can reject the Kantian position about the science of logic, but support the idea that there are laws of reasoning which do not change. And even if we support the biological theory of evolution, considering that human beings are biologically changing, this does not justify the evolution of reasoning in the last 10.000

years, because biological speaking there were not significant changes in human beings these last 10.000 years [30].

Human culture is very various and is changing, and one may want to argue that logic is a cultural phenomenon like any other, such as politics, music and language. However let us point out that the very idea of science which has emerged these last 2.500 years does not correspond to a phenomenon related to a specific culture and that it has spread all other the world, it has a universal dimension.

As Hilbert emphasized "mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one county". This does not imply that mathematics is not changing or/and that mathematics is describing a reality which is not changing. We will not here enter in the details of the history of mathematics, but we can stress that results which were proven 2.500 years ago in a specific region of the universe, like the infinity of prime numbers, are still considered as universally valid despite important changes in the context, language, framework [45].

The relation between the science of mathematics and its object of study is clearly interactive. One can claim that the situation is the same with logic. But logic is not at the same level as mathematics, it is above mathematics, as suggested by the terminology promoted by Hilbert "metamathematics":

> The axioms and provable propositions, that is, the formulae which arise in this interplay, are the representations of the thoughts which constitute the usual procedure of the previous mathematics, but they are not themselves the truths in the absolute sense. The absolute truths are rather to be regarded as the insights which are provided by my proof theory, namely the provability and the contradiction of these formula systems. [30]

Hilbert also called metamathematics *proof theory*, because the objects of study are mathematical proofs. As Bourbaki claims at the beginning of his monumental work [28]: *Qui dit mathématique, dit demonstration*. However, one may think that mathematics does not reduce to proofs. And nowadays the logic tree has grown and proof theory is just a branch of logic. Another central concept of modern logic is the concept of *truth*. Proof and truth are in fact like brother and sister, the relation between the two has been dramatically depicted by Alfred Tarski [44], the originator of the drama being Kurt Gödel, who, with his famous incompleteness theorem,

inspired by the liar paradox, has tragically broken the union of the two forever.

This is the right time to remember that truth can be considered as the heart of logic in the same way that goodness is considered as the heart of ethics and beauty the heart of esthetics. If one claims that logic is relative, does this mean that truth is relative? Sophists already tried to relativize truth. With anti-exceptionalism, are we back to sophistry?

Tarski's theory of truth does not relativize truth, it is just a precise and formal presentation of a theory directly inspired by Aristotle. The notion of truth is used in many different ways in modern logic. But this is not necessarily a *hard* relativization, it is more like a spreading, the growth of a tree.

Talking about trees, we can see the things upside down, logic not as above but as the root, the foundation of mathematics, and more generally the foundation of science. The terminology *Logic and Methodology of Deductive Science* was promoted by the Polish school , which can be seen as corresponding to a program extending the project of metamathematics of Hilbert [12]. Another terminology promoted by the Polish school is "Metalogic", as a general theory of logical systems [14].

6. The exceptional interplay between logic and metalogic

The difference between *logic* and *metalogic* is not the same one we have made between reasoning (*Logic*) and the science of reasoning (*logic*). That's another difference! In a given system of logic there are *theorems.* For example, in classical logic the following is a theorem:

$$\neg(p \wedge \neg p) \qquad \text{(TNC)}$$

A *metatheorem* is a theorem about the system, like the *replacement theorem*, saying that we can replace a proposition by a logically equivalent one or *Post maximality*, saying that we cannot strengthen the connectives of classical propositional logic (CPL), that therefore CPL is maximal.

Metatheorems are clearly part of the science of logic. But theorems are also part of the science of logic. They are parts of a system of logic which supposedly describes reasoning, at best mirror it, but they are not reasoning itself. For example, the above theorem (TNC) of classical logic is not a principle of reasoning as reasoning in action, at best an account of it.

The *principle of non-contradiction* (PNC) has been considered, and is still considered, as one of the basic principles of reasoning. It is important to make the distinction between PNC as part of reasoning (*Logic*), and formulations of it in a given system of logic or in the theory of this system (its metalogic). The science of logic permits to study PNC, to answer in some way the question: what is PNC?

PNC is multifaceted, there is not an absolute true version of the PNC. PNC in this sense is relative. But we have to distinguish this relativity from the relativity of the object itself. This is not because we cannot really capture it, that it is necessarily relative. Also, this relativity of understanding PNC is different from the relativization of the PNC from the point of view of systems violating it, the so-called "paraconsistent logics". But the two are interrelated.

It can be shown in the framework of modern logic that (TNC) is not equivalent to

$$p \wedge \neg p \longrightarrow q \qquad \text{(EC)}$$

or to

$$(\,(\neg p \longrightarrow q) \wedge (\neg p \longrightarrow \neg q)) \longrightarrow p \qquad \text{(RA)}$$

and moreover that (RA) is not equivalent to:

$$((p \longrightarrow q) \wedge (p \longrightarrow \neg q)) \longrightarrow \neg p \qquad \text{(RAW)}$$

But it is also possible to prove, at the metalogical level, that from (RA) we can derive (RAW), (EC) and (TNC) and indeed all properties of classical negation (see [2]).

The theoretical study of (PNC) leads to the creation of systems describing partial aspects of it, that can be implemented as new forms of reasoning. This is a practical counterpart of the interaction between logic and metalogic. There is also a more philosophical or/and fundamental counterpart. For example, Boole [26] has shown than in some sense we can derive (PNC) from

$$p \wedge p = p$$

For details see [13].

Let us now examine the following statement:

(SCN) The classical negation of a proposition $\neg p$ is *false* if and only if the proposition p is *true*.

Is it PNC? First let us note that this is not a theorem of a system of logic. It is a principle defining negation that corresponds to the behavior of classical

negation in a given system. It is usually considered as the semantics of classical negation, using two "true-values", *truth* and *falsity*.

What is the difference between that and the following:

> (CT) A proposition is *contradictory* to another one if and only if they cannot both be true together and cannot both be false together.

One important difference is that in (CT) there is not a connective of negation. We can say that, according to the semantics of classical negation, p and $\neg p$ are contradictory. There is the question to know if in the case we have a negation such that p and $\neg p$ are not contradictory, we can still say that the connective "¬" is a negation [8]. A good reason to reply positively to this question is the theory of opposition, according to which, besides (CT) we have two other oppositions, *contrariety* and *subcontariety* [3].

The theory of opposition goes back to Aristotle and the square of opposition [23]. This theory can be considered as part of metalogic. It has numerous applications, helping to clarify our reasoning, improving it. The square of opposition was first used as a metalogic framework for syllogistic categorical propositions. But the theory has evolved. In particular in the 20th century was developed mainly by Robert Blanché the hexagon of opposition [25]. This hexagon has in particular permitted to clarify the relation and distinction within three families of concepts, quantifiers, alethic modalities and deontic notions, explaining the differences between some and there exits (Fig. 3), possible and contingent (Fig. 4), permissible and optional (Fig. 5).

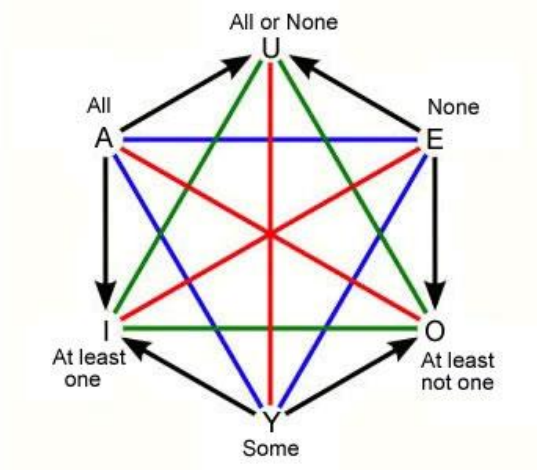

Fig. 3 The Hexagon of Quantifiers

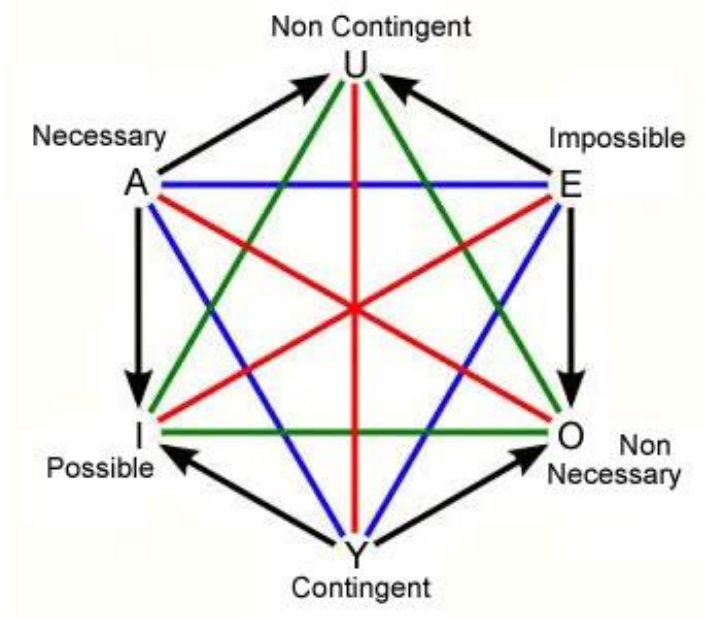

Fig.4 The Hexagon of Alethic Modalities

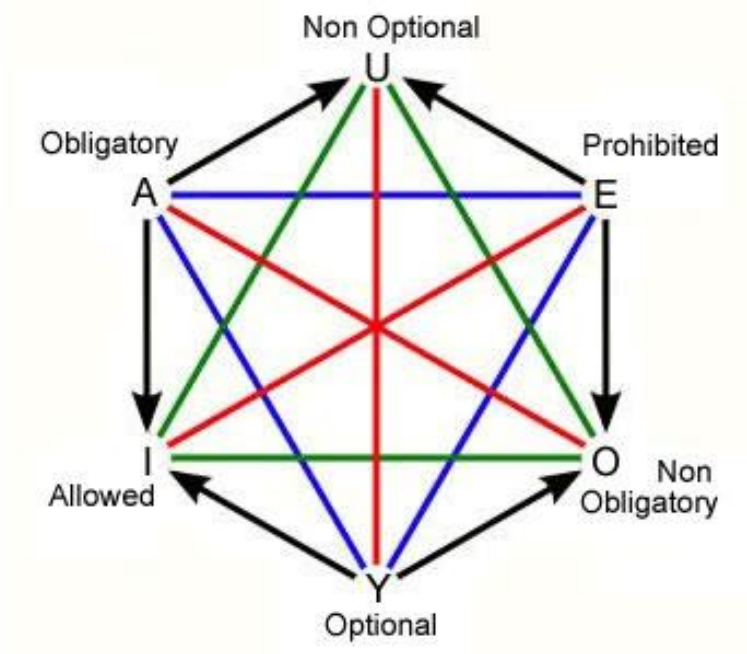

Fig.5 The Hexagon of Deontic Modalities

Moreover, the hexagon of opposition can be used to depict the six positions a proposition *k* may have vis-à-vis a theory *T* [19] (Fig 6.):

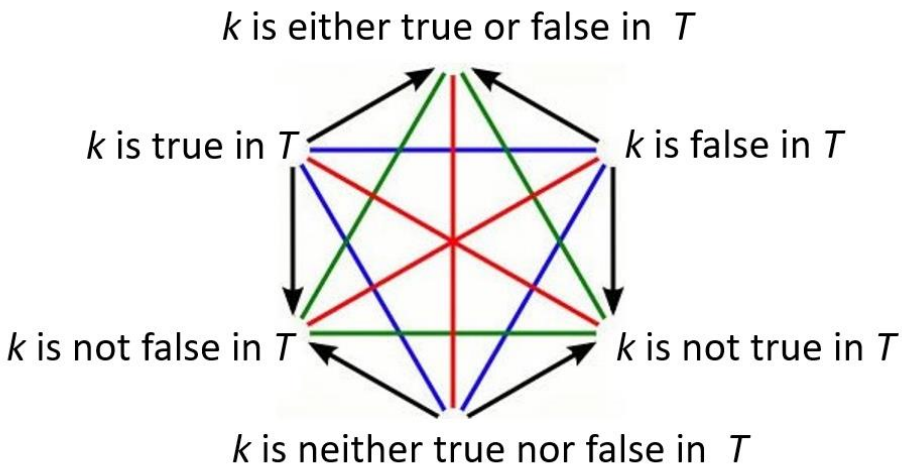

Fig.6 The Hexagon of Proposition/Theory

The hexagon of opposition can also be applied to the theory of opposition itself (Fig. 7):

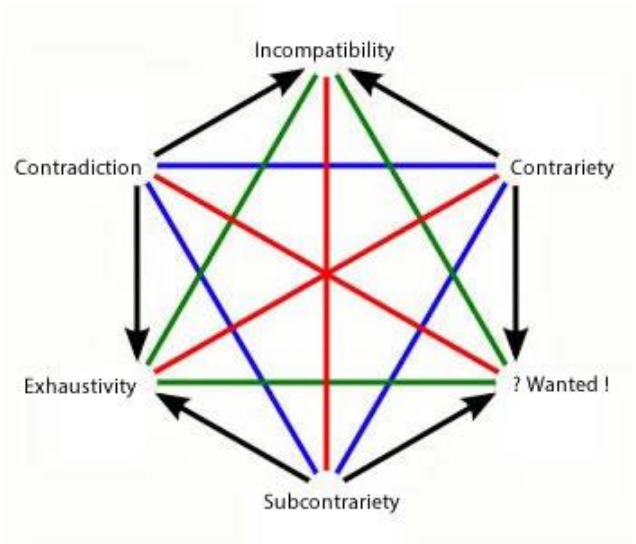

Fig.7 The Hexagon of Opposition of Oppositions

These are striking examples of interaction between *logic* and *metalogic* showing how the science of logic is exceptional.

7. The exceptional interdisciplinarity of logic

Can we say that logic is exceptional because it is more interdisciplinary than any other discipline?

"More" can be interpreted here from the point of view of *quantity*, the number of disciplines with which logic is interacting, and *quality*, the deepness of the interaction. And also we are talking about logic not only as a science, but also as reasoning, as an activity. Interdisciplinarity of an activity means interaction with other activities. We can say for example than dance is deeply interdisciplinary with music.

Interaction is not just application. Interaction must benefit both sides. Mathematics applies to finance but how much benefit does it get from finance, if any?

In logic there is the important distinction between the form and content of reasoning. From the point of view of the content, reasoning is "in touch" with any rational activity. We can properly speak about interaction when the form of reasoning will properly shape a given field and when this field will itself shape reasoning, generating new forms of reasoning.

From this perspective, the interaction between syllogistic and mathematics is of degree zero not to say below zero … However, logic does not reduce to syllogistic and mathematics does not reduce to zero. The interaction between logic and mathematics is important. Let us examine two pivotal complementary examples: *reductio ad absurdum* (RA) and *Boole's logic* (BO). (RA) is first of all a reasoning, on the side therefore of *Logic* and (B0) is part of the science of reasoning, therefore on the site of *logic*.

The reasoning by the absurd was imported into mathematics with a first spectacular result, the proof of irrationality of square root of two. Some *très chic* people like Jean Dieudonné think this proof is indeed the birth of mathematics [32]. Most of the proofs of mathematics are using (RA). This tool has really shaped mathematics, in particular it has changed the general landscape of mathematics, introducing in the scenery irrational numbers. On the other hand, modern mathematics using tools from mathematics has permitted to have a better understanding of the very nature of (RA), distinguishing various versions of it and forms of reasoning close to it (see previous section).

Regarding (BO), it is well-known that Boole has been directly influenced by the British school of symbolic algebra to give a new perspective and new

understanding of syllogistic. So mathematics has really shaped the theory of reasoning. What is less known or/and emphasized is that (BO) has also seriously changed mathematics, raising it to a more abstract level, where objects can be anything not only numbers, in particular propositions. As Mary Everest Boole said : “Many people think that it is impossible to make algebra about anything except number. This is a complete mistake … The use of algebra is to free people from bondage” [27]. Mathematics is exceptional in many senses; it is not a science like any other and interaction between logic and mathematics is exceptional.

Let us examine another “matter” : philosophy. One who knows a bit of history can hardly claim that philosophy is just a discipline like other ones. It is enough to remember Plato’s academy and Descartes’s tree. According to the legend, was written at the entrance of Plato’s academy “no one enter here if he knows geometry”. This stance established a triangle between mathematics, logic and philosophy, which is the key to science and human understanding [8].

Plato has developed philosophy based on reasoning, inspired by mathematical reasoning, but getting higher, closer to truth. Logic can be applied to philosophy not just as reasoning, through argumentation, but also as the science of reasoning, developing useful structures that can help to have a better understanding of fundamental philosophical notions such as the double dualities put forward by Kant, that can be explained and/or clarified by the following hexagon of opposition (Fig 8.):

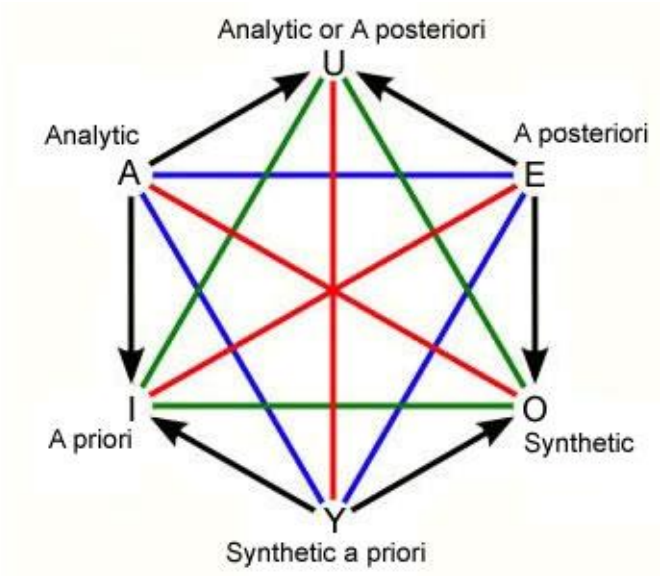

Fig.8 The Hexagon of Opposition applied to Kantian Philosophy

Besides philosophy and mathematics there is a third important interacting activity with logic, it is computation. The theory of computation was developed within the science of reasoning. One of the most impressive results of modern logic is a theory explaining the distinction and relation between proof and computation. The theory of computation has changed our way to understand what reasoning is, the way we are reasoning and our daily life …

If we consider the interaction between logic, philosophy, mathematics and computation, logic appears as truly exceptional. But there are also other fields like semiology, theology, physics, economics, music, color theory, etc.

8. The transdisciplinarity of logic : from *mathesis universalis* to *logica universalis*

Transdisciplinarity was promoted mainly by Jean Piaget (1896-1980) (see [38]). What is transdisciplinarity? To answer this question, we may ask: who was Jean Piaget? Or: what kind of man he was? He is mainly known as a psychologist, an epistemologist and an educational theorist, but he also wrote a logic book, *Traité de Logique* (1949) [39]. And besides a classification by fields/disciplines, what is worth emphasizing is that a main theme of Piaget's research is the development of intelligence and there is a connection between intelligence and logic in its double sense.

From the perspective of Piaget, transdiciplinarity is not just an interaction between different disciplines but something new emerging and directing this interaction, breaking the bounds. By its very character, transdisciplinary is not a new discipline, but something which is at another level, above specific disciplines. What is the nature, characteristics and name of such transdisciplinary level?

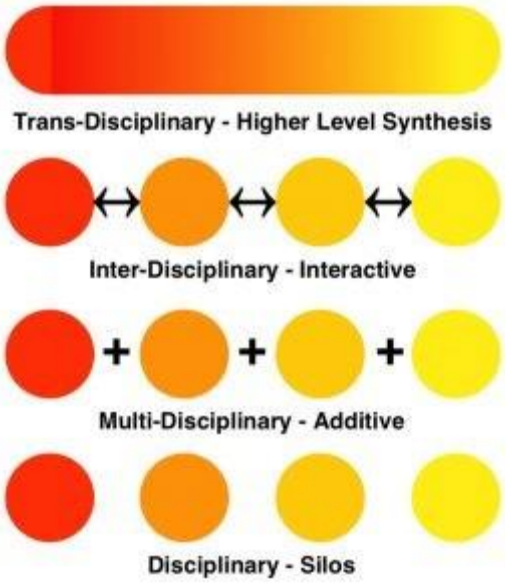

Fig.9 From Disciplinary to Transdisciplinary [25]

Descartes's *Mathesis Universalis* is going in this direction. Descartes was using the expression "mathesis universalis" to talk about a general science:

> … there must be some general science which could explain all that which can be investigated concerning order and measure irrespective of any particular matter. And I realized that this science should be designed not with a far-fetched word, but by an already venerable term with a received usage, as "mathesis universalis", since it contains all what by virtue of which the other sciences are also called 'part of mathematics'. [31]

This is at the end of rule 4. In rule 5 Descartes goes further at a superior level saying that:

> The whole method consists in the order and arrangement of the things on which the vision of the mind has to be focused in order that we might discover the truth. And yet we shall be following this method exactly if, step by step, we reduce complicated and obscure propositions to simpler ones and we then try to ascend, through the same steps, from intuition of the simplest ones of all to a knowledge of all the others. [31]

In the methodology of Descartes there is a double aspect: on the one hand a rather abstract approach to reality based on mathematics, on the other hand a logical articulation of this knowledge [41]. By contrast, Leibniz's methodology that he also characterizes using the expression "mathesis universalis" reduces to the form of reasoning [42]. More

interesting and closer to Descartes is the development of modern mathematics where we find a similar articulation between abstraction and reasoning.

The abstraction of modern mathematics is higher than the one of Descartes's *Mathesis Universalis*, but this is in some sense a continuation of it. The fundamental notion of modern mathematics is the notion of structure, and structures of order play a fundamental role, in particular lattice theory. At some point the word "structure" was used as synonymous of "lattice" [33]. Lattice theory is a symbolic bridge between logic and mathematics, since lattices are used to describe reasoning.

The relation between structures and logic was fully developed by Alfred Tarski. With Lindenbaum he characterized logical notions as those which are invariant under any transformation (see [15], [37] and [49]) . Later on, he developed model theory, which is the latest theory of the four which constitutes modern logic, after set theory, proof theory and recursion theory [47].

If we consider these four theories it appears clearly that modern logic is at the highest transdisciplinary level, the product of an interaction between semiotics, computation, mathematics and philosophy. For this reason, it is better to use the expression *Logica Universalis*, than *Mathesis Universalis*.

This expression was used in particular by analogy with *Universal Algebra* (*Algebra Universalis*) [1]. There is a fundamental aspect in this analogy which changes the rules of the game, the fact that there are no axioms anymore (cf. [5], [20]). It does not mean that the notion of axiom is put aside, but that there are no absolute axioms [17] [18]. This is of course clear in the development of modern axiomatic. But this was not clear for logic itself, people wanting to construct absolute systems, two famous examples being Whitehead and Russell in *Principia Mathematica* [50] or Stanisław Leśniewski [36].

But Tarski, the only PhD student of Leśniewski, already started to change the situation with other colleagues of the Polish School by developing the theory of consequence operator, already a much more abstract theory of reasoning, but still with axioms [46].

Model theory is exceptional in many different ways, in particular through its methodology (interaction between language and interpretations) and through its applications (which go far beyond mathematics itself). Universal logic is the result of applying model theory to logic itself [15].

9. The universe, the universal and the universality of logic

There is universality in the sense of a *universal stuff* (e.g. *a universal language*), and universality in the sense of the *universe*.

The universe is a global notion, not reducing to the globe, but sometimes with a particular sense, relative to a specific field, like the universe of discourse, the universe in set theory. When one thinks about the universe, (Fig. 10) is more or less what one has in mind, something astronomical.

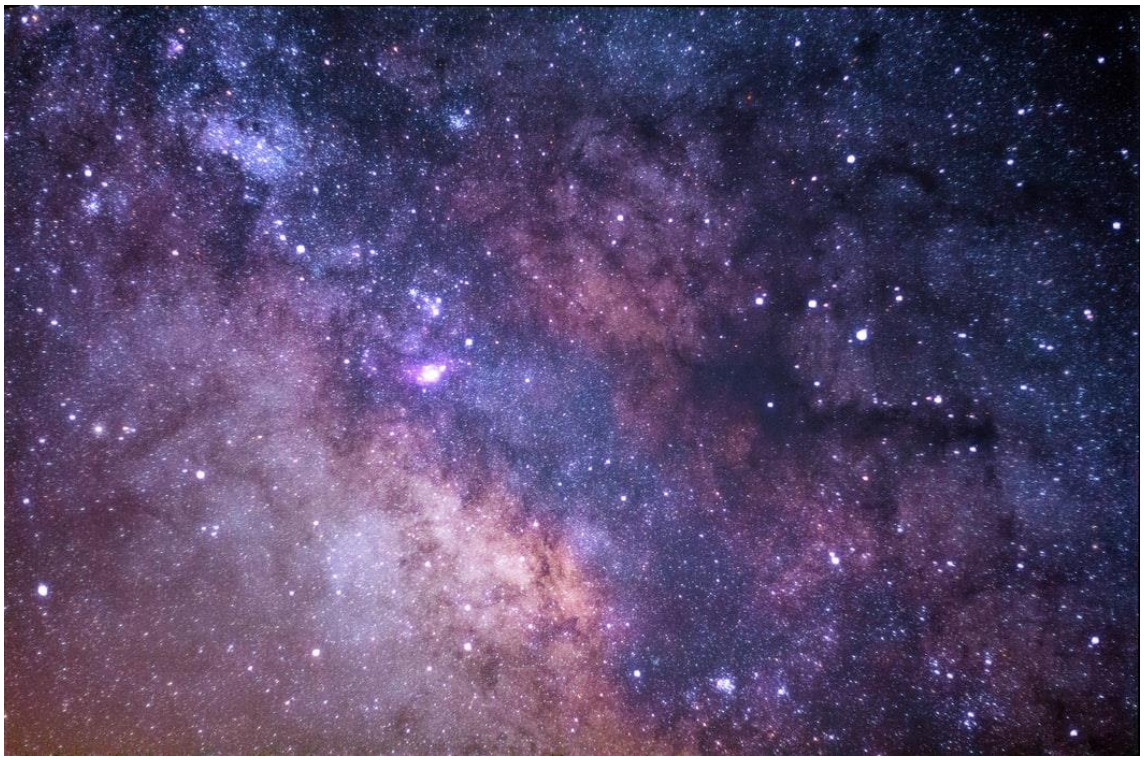

Fig.10 An astronomical vision of the world
(Jeremy Thomas / Unsplash)

This image corresponds to a totality which is basically conceived as physical. This view is macroscopical, but this not necessarily opposed to a microscopical view, if one imagines the microscopical world in an atomistic perspective in particular having in mind the Rutherford-Bohr's atom. This vision is predominantly physicalist for two reasons: up to now the only place where life has been located is on a very small part of the universe, the earth, there is a tendency in science of physicalism, in particular to reduce

biological phenomena to physical phenomena, through the famous soap history.

The *universe* is the Latin version of the Greek *cosmos*, according to which "all this" is organized, there is a certain order. This orderly ontology is a basic and necessary presupposition for physics and science in general. This can be expressed in a more "metaphysical way", using the principle of sufficient reason: *nihil est sine ratione*, emphasized by Leibniz. The literal translation of this principle in English is *Nothing is without reason*. This principle is something logical … Nothing is without reason can be considered as the most fundamental principle of reality. And the fact that human beings are rational (logical) animals explains why they are able to understand the world, the key to Einstein's enigma: "The most incomprehensible thing about the world is that it is comprehensible." From this double rational universal perspective, objective and subjective, logic is really exceptional.

When climbing at the metaphysical level, things get less physical. In the objective sense, because everything has a reason applies also to biological, psychological, sociological, anthropological phenomena. And in the subjective sense, unless one wants to reduce both reasoning and the science of reasoning to a physical phenomenon through firstly the reduction of logic to a biological phenomenon and secondly this biological phenomenon to a physical phenomenon.

Now let us examine the sense of universality of logic from the point of view of the universality of reasoning. Someone may argue that we may need different types of reasoning depending on the situation. But this is not an argument against the universality of logic, unless one believes that there is a universal system of logic solving all the problems [10]. This is not the perspective of universal logic, especially emphasizing the difference between reasoning and the science of reasoning.

Reasoning finds its way among the universe of all things including reasoning itself.

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