

A hexagon of opposition to clarify Kant's two dichotomies *analytic / synthetic - a priori / a posteriori*

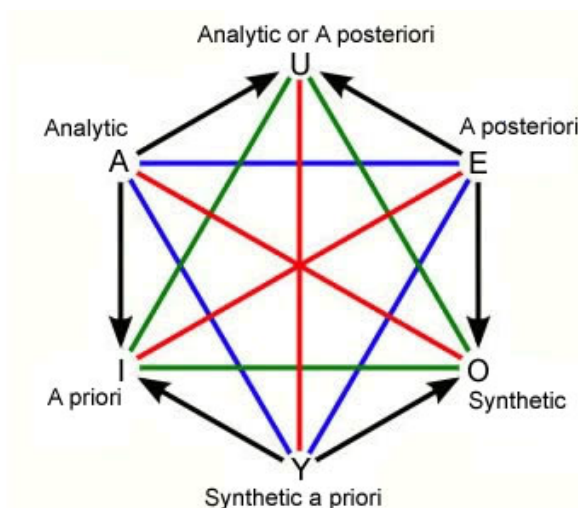
Jean-Yves Beziau

Dedicated to Jean Ferrari (1931-2024). Founder and president (1988-2017) of *Société des études kantienne de langue française* and president of ASPLF (1996-2010) *Association des sociétés de philosophie de langue française*, who encouraged me to organize the 37th edition of the ASPLF congress in Rio de Janeiro in 2018, on imagination.

Abstract. We present an analysis of Kant's double dichotomies *analytic / synthetic - a priori / a posteriori* based on Blanché's hexagon of opposition. We start by a general discussion about what an interpretation is and the difference between philosophy and science, emphasizing that our approach here about Kant's philosophy is rather scientific. We then explain how the scientific tool we are using, the hexagon, works and why it is a good tool in general to answer the socratico-platonic question "What is it?". We also discuss the question of the *a prioricity* of the hexagon, criticizing Kant's position, according to which only dichotomy is *a priori*. We then use the double pair *good/bad – beautiful/ugly* as a guinea pig for developing hexagonal considerations about dichotomies. After that, we apply this methodology to Kant's double pair exhibiting four hexagons and showing that only one corresponds to Kant's promotion of the *synthetic a priori*. We explain how this hexagon can be interpreted and justified.

Keywords. Kant; analytic/synthetic; a priori/a posteriori; square of opposition

Mathematics Subject Classification (2010). Primary: 03A05 Secondary: 03B42;03B45



1. Interpretation and understanding

We give here an *interpretation* of Kant's famous double pair *analytic/synthetic - a priori/a posteriori*. For that, we use the framework of the square of opposition.

An interpretation can be a way of bringing understanding, clarifying the situation, but hermeneutics can also be hermetic or confusing. One may get lost in a jungle of interpretations. There can be different ways of interpreting the same thing, or not ...

In *Model Theory*, one of the four main branches of modern mathematical logic, together with *Set Theory*, *Proof Theory*, *Recursion Theory*, a theory is called "categorical" when all its interpretations, its *models*, are the same. A theory can have no model, when it is inconsistent. A theory having at least one model is said to be consistent. An inconsistent theory must be differentiated from a non-categorical theory having different models incompatible with each other. For example, the general theory of order is based on two axioms: anti-symmetry and transitivity. This theory has models with a first element and models with no such an element. These models are incompatible, but the theory of order is not inconsistent. There are also extensions of the theory of order which are categorical: for example, all the denumerable models of dense order without first and last elements are isomorphic to the ordered structure of the rational numbers.

Does it make sense to ask if Christian religion is consistent or inconsistent, categorical or non-categorical? It depends if we can consider it as a theory or not, if it makes sense to examine the *Bible* as a kind of theory that can be interpreted. The same apply to philosophical doctrines.

There is an important difference between what today is called "philosophy" and what is called "science". When studying philosophy what generally predominates are *philosophers*, even in analytic philosophy (cf. Frege, Russell, Wittgenstein, Kripke), in the case of science, *theories* (e.g. number theory, the theory of evolution, the theory of relativity). In both cases there are no absolute truths, but for different reasons.

In philosophy, it is impossible to say that such or such interpretation/presentation is exactly what a given philosopher, e.g. Kant, was thinking. In science, theories are changing and evolving all the time. The main point is not to understand the theory of relativity, relatively to Einstein original thought, but to examine the truth of this theory, on the one hand relatively to its own consistency, on the other hand relatively to reality. The theory is changing and probably we will never know what reality really is. Moreover, there is no *Bible of the Theory of Relativity*. It cannot be perfectly and definitely fixed in a given book.

In philosophy, one may want to stick to the idiosyncrasy of a philosopher or rather to study his/her ideas in a more scientific and objective way. These different approaches depend on the philosopher. Kant is a philosopher who facilitates a scientific approach because his ideas can be seen as quite theoretical. This is the case of the double pair we are talking about here.

Our goal is not to reach the exact original Kant's understanding, but to develop an understanding based and/or inspired by his theoretical ideas. Some people have argued that Kant's idea of *synthetic a priori* was wrong or nonsense. Here we present an interpretation where this idea is consistent and makes sense. Even if it is not exactly what Kant had in mind, and this we will never know, we nevertheless think that this interpretation will not wake up Kant in his tomb in Königsberg. Nevertheless, this is not the final point of the discussion which can continue until the end of time.

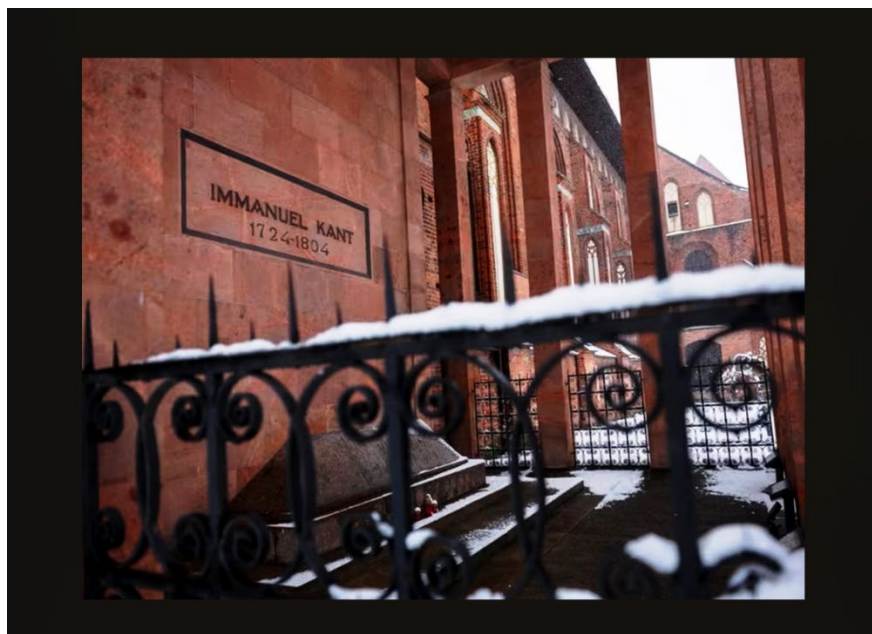


Fig. 1 - Kant's tomb in Königsberg

2. Squaring Kant's double pair

We have used the word "pair" to be as neutral as possible. We will interpret these two pairs as two *dichotomies*. Dichotomy is a very general idea that can be presented in an intuitive / symbolic way or in a more formal logical way. But the two approaches in this case are directly connected without much ambiguity.

We are using the framework of the square of opposition which is indeed something at the middle of intuition and formalization, this is why we are using the word "framework" rather than "theory", and also because it is a meta-theory rather than a theory.

Modern logic is sometimes called “formal logic” to distinguish it from traditional pre-Boolean logic, but this is quite ambiguous because, as we have pointed out (see [1]), the expression “formal logic” has five different meanings and Kant himself is considered the one who introduced or/and promoted the expression “formal logic” (as pointed out by H.Scholz [32]). To say that we are here formalizing Kant’s theory would be rather ambiguous. We prefer to stay that we are *squaring* Kant’s theory.

Modern logic is also called sometimes “symbolic logic” (cf. *The Journal of Symbolic Logic* and the *Association for Symbolic Logic*), but this is highly ambiguous, because the word “symbolic” is there neither use in its original etymological semiotic sense or in the Venn sense, John Venn being the guy who coined the expression “symbolic logic”. Before Venn, people, even if they were not using the expression “symbolic logic”, were using diagrams in logic, in particular Euler, Kant himself, and Schopenhauer, used diagrams.

And the most famous logic diagram is the square of opposition. We are using the expression “framework of the square of opposition”, rather than “square of opposition” *tout court*, because this framework does not reduce to the most famous diagram illustrating this framework. This framework is beyond the square diagram which does not reduce to this diagram. It started to be developed by Aristotle before the square diagram was produced (by Apuleius and Boethius) and many diagrams other than this square were later created within this framework (see [11], [14], [15]). The most famous one is the hexagon of opposition of Robert Blanché that we will use here (see [17]).

The hexagon of Blanché is not a trivial generalization of the square of opposition to a polygon with more than four sides, it is a reconstruction of the basic square of opposition giving full meaning to it (see [2]). And the hexagon has the quality of being perfectly symmetric encompassing three squares of opposition:

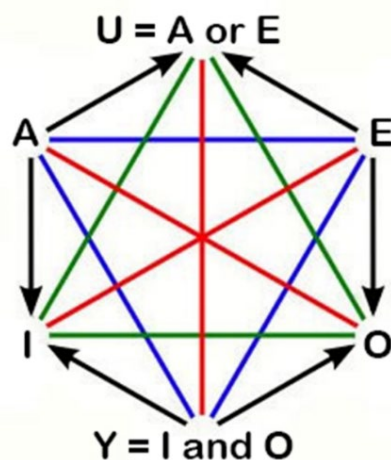


Fig. 2 – Blanché’s hexagon of opposition

3. A priori, dichotomy and polytomy

Kant had the idea that only dichotomy was *a priori* and that polytomies (bigger than 2) were *a posteriori* (see [24]). Something like the dichotomy *Good and Bad* is for example *a priori*, on the other hand the theory of the four elements made of *Earth, Wind, Fire, and Water*, is an *a posteriori* quatritomy.

But let us point out that in both cases we have *exclusion* between the different sides: something which is good is not bad and vice versa; earth is not water and water is not earth (although they can be mixed, giving birth to clay), earth is not fire and fire is not earth, etc. And also, in both cases, we have *exhaustion*: something cannot be neither good, nor bad; something cannot be neither earth, nor wind, nor fire, nor water. So, dichotomy and quadritomy, obey the same rules, and this is true for any polytomy.

When we go beyond dichotomy, following the principles of exhaustion and exclusion, contradiction turns into contrariety, a notion put forward by Aristotle himself. *Obligation* and *prohibition* are not contradictory notion, but contrary notions, because something can be neither obligatory, nor prohibited: it can be *optional*. We have therefore the following deontic contrariety triangle:

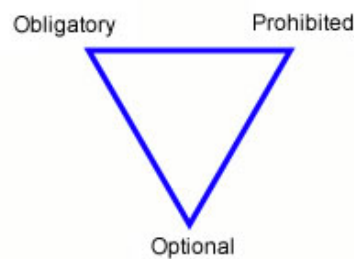


Fig. 3 - The deontic contrary triangle

On this basis, we can construct the following hexagon of opposition, which allows to clearly distinguish between optional and allowed:

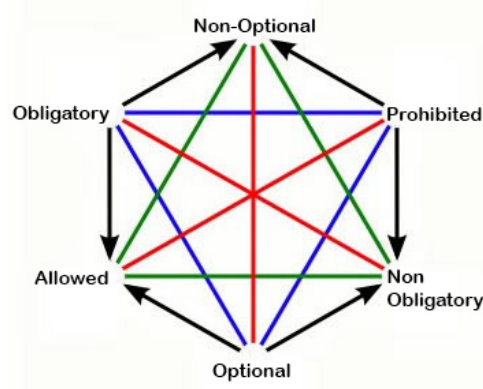


Fig. 4 - The deontic hexagon

In the same way Blanché was able to distinguish contingency from possibility building a hexagon of alethic modalities:

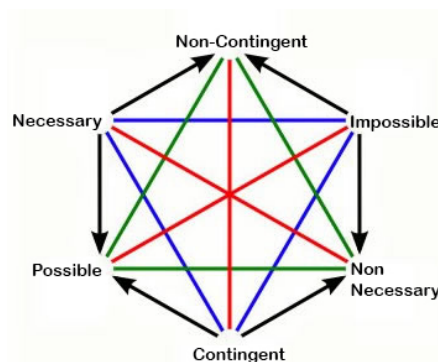


Fig. 5 - The hexagon of alethic modalities

Blanché had the idea that trichotomy was central. Based on the many examples of trichotomies we could argue that trichotomy is *a priori* and/or more natural than dichotomy. Sometimes dichotomy seems indeed a bit artificial. If quadritomy is *a priori* and/or natural, that is a different kettle of fish, that we will leave for the next square banquet.

What we can say is that we can develop a theory of *n*-opposition obeying the principle of exclusion and exhaustion. The next step after the square, is the hexagon and then we go to the third dimension (see [4]). The formal theory of *n*-opposition is indeed rather again the idea that only dichotomy is *a priori*.

The fact that dichotomy itself is *a priori* or not is an interesting question, related to the story of the snake that bites its own tail, the concept of *a prioricity* being applied to the dichotomy *a priori*/*a posteriori*.

One may want to break the dichotomy *a priori* / *a posteriori* and go to a triangle of contrariety. But it is not because we have a general framework for opposition going beyond dichotomy that every dichotomy should be broken. Nevertheless, a dichotomy can be better understood relating it to other dichotomies, putting it in a square / hexagon of opposition. For example, instead of breaking the dichotomy *identity* / *difference*, we can put it in the following hexagon (cf. 5):

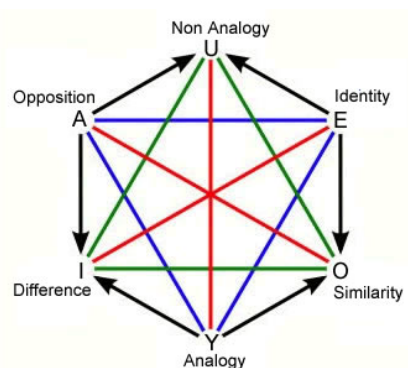


Fig. 6 - The analogical hexagon

Designing a figure of opposition is a way to understand the notions involved in it, by relating them, following a structural approach promoted by Blanché and the neo-Saussurean school.

Given a notion like *analogy*, we can ask the socratico-platonic question: *What is Analogy?* Plato made a dichotomic opposition between comprehension and enumeration (cf. the famous example of the clay in the *Theaetetus* [30]) that can be presented in the table below.

NORTH POLE	SOUTH POLE
Enumeration	Comprehension
Description	Definition
Extension	Intension

Fig. 7- Bipolar table for the question “What is X?”

But we may defend an Equatorial position, especially if we believe that there is no essence of things, in particular of “something” like analogy, which is a human’s idea floating in time and place. The Equatorial approach is going beyond the dichotomy *North pole / South pole* (see [7]). And a way to do that is to use the theory of opposition, in which dichotomy is still there, but complemented by other relations of opposition. We recently did that, besides analogy [5], for silence [6], movement [8] and symmetry [9].

Plato was indeed himself using dichotomy, inspired by Pythagoras and his table of opposites, but dichotomy in an abstract way. The dichotomies of Pythagoras are rather empirical. To turn dichotomy into an *a priori* abstract principle is related to negation, the principle of non-contradiction and the reduction to the absurd, which led to the refutation of the Pythagorean dogma according to which all numbers are natural numbers or relations between them (*rational* numbers).

4. The good, the bad, the ugly and the beautiful

A way to understand a notion is through a dichotomy and then relating this dichotomy to another dichotomy.

Before dealing with the main dish of our paper, the two dichotomies *a priori/a posteriori* - *synthetic/analytic*, we consider, as an appetizer, a tasty guinea pig made of the two fashionable dichotomies *Good/Bad*, *Beautiful/Ugly*. Let us cross them into squares within hexagons:

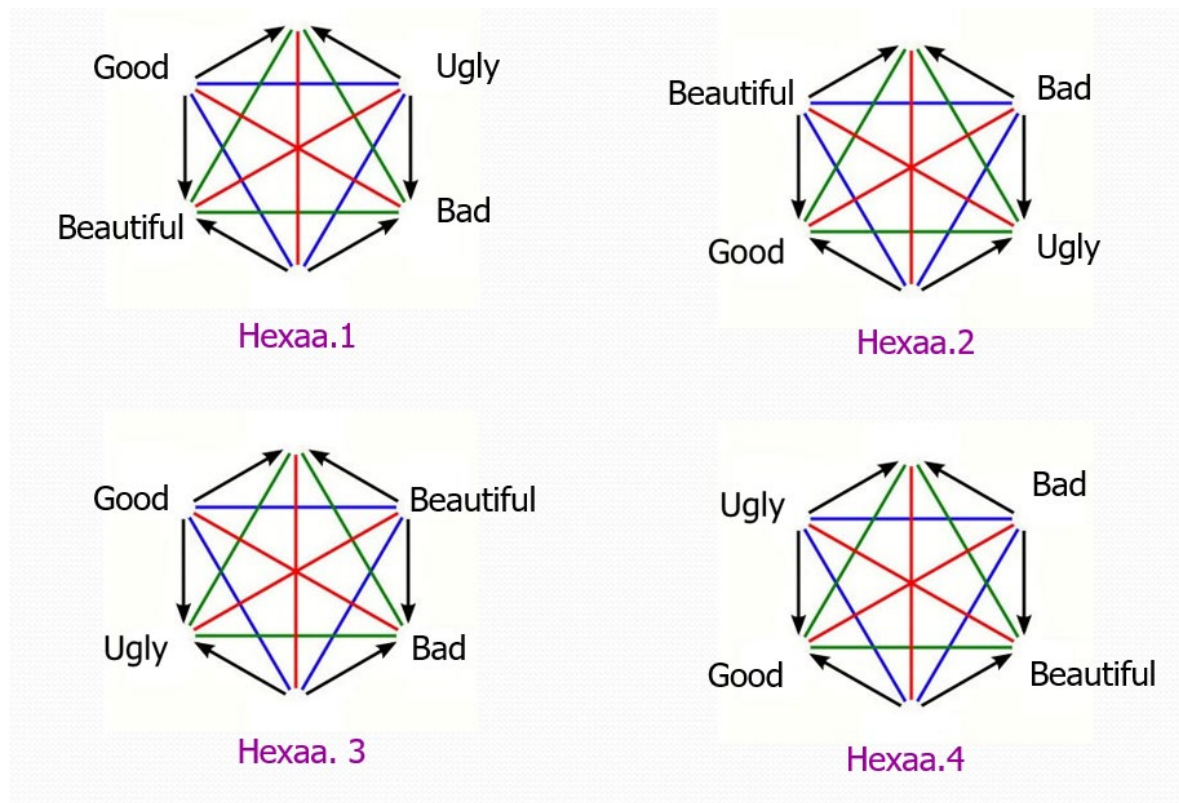


Fig. 8 – Four hexagons for the dichotomies *good/bad* – *ugly/beautiful*

Which one is the best? Everything is possible! And these are exactly the four possibilities. What does this mean? It is similar for example to a Venn diagram (see [10]) or the table of the 16 connectives of classical propositional logic (see [3]). It is a visual description of a systematic rationalization of a situation. We can say that even if it is not absolute, it something fitting well in the *a priori* realm by contrast to the *a posteriori* one. At this stage we don't have to wander in the jungle to find the philosophical stone. But we have some choice in the representation of this rationalization. Venn diagram is a very nice way to represent the seven possibilities of interaction between three notions and its success is due to this striking visualization. The table of the connectives is also pretty good, but perhaps less attractive. Blanché's hexagon, especially in its colorful version is more impressive, and it can also be applied to connectives (see [16]).

We can exclude Hexaa.3 and Hexaa.4, where respectively *Good* implies *Ugly* and *Ugly* implies *Good*, because this is not really politically correct, although the bottom Y-corners of these two hexagons, respectively *Ugly* and *Bad* and *Good* and *Beautiful*, are solid bases for successful Hollywood movies.

We are now left with the following two hexagons:

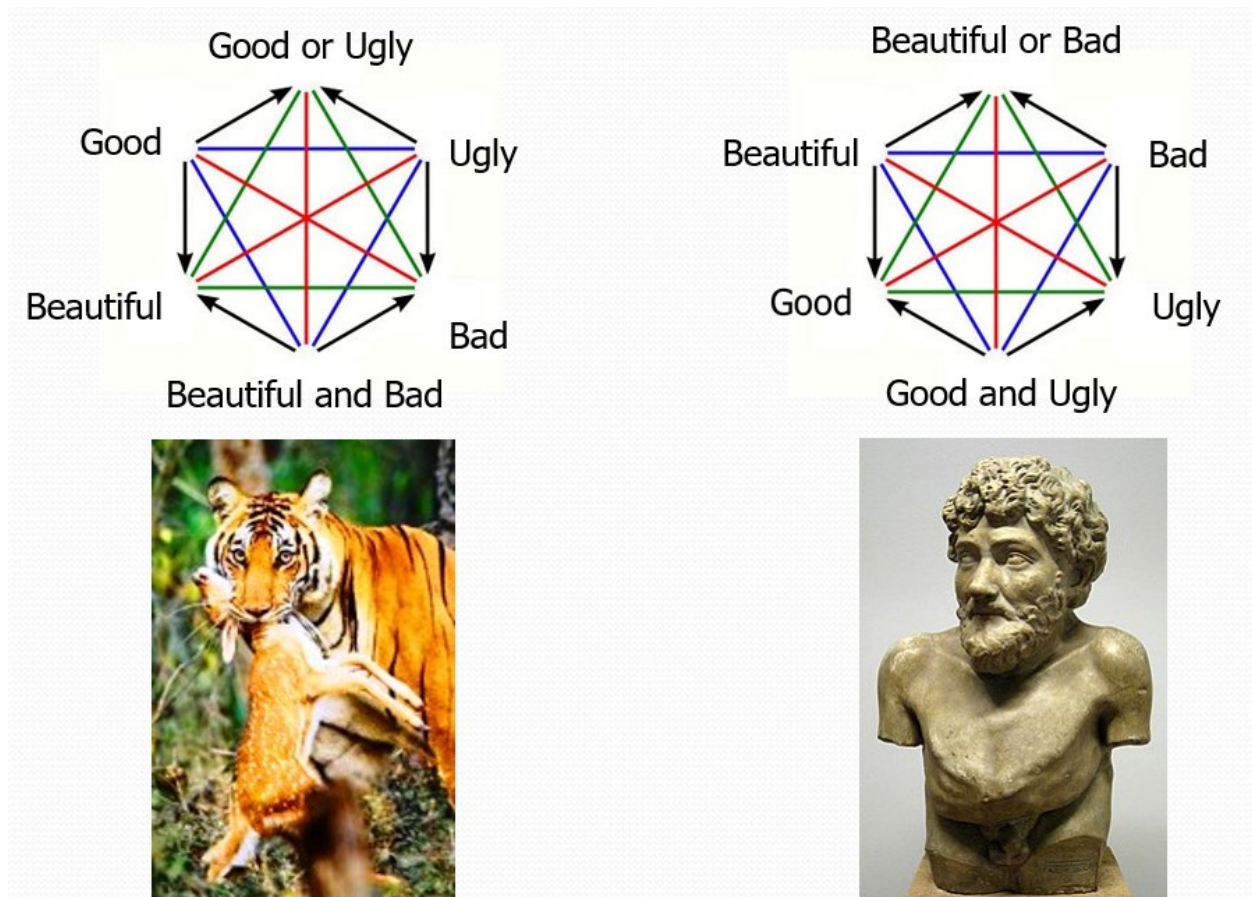


Fig. 9 – The two valuable hexagons to put together the dichotomies *good/bad - ugly beautiful*

We must choose between goodness as a particular case of beauty (left) or beauty as a particular case of goodness (right). Both cases seem reasonable. The second one is more platonic if we consider that Agathon is the Queen.

The problem is now with the two Y-corners! We must choose between *Beautiful and Bad* and *Good and Ugly*. It is a bit like having to choose between Charybdis and Scylla. The right hexagon looks more friendly, considering the wisdom of Aesop (“A liar will not be believed, even when he speaks the truth”) and the fact that this hexagon is platonic.

5. A hexagon of opposition relating the two Kantian pairs

Here are the four possibilities to put the two Kant's dichotomies in a square within a hexagon of opposition:

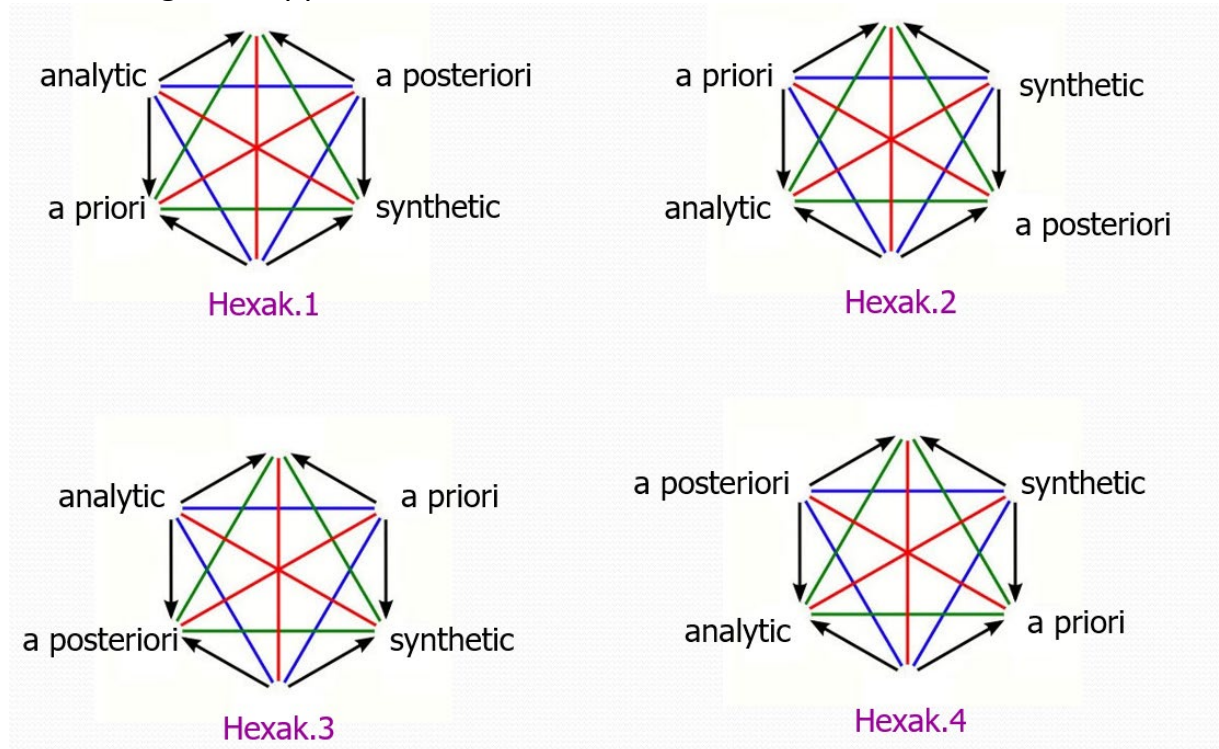


Fig. 10 – Four hexagons for Kant's pair of dichotomies

To analyze the situation and to decide which one of these figures is the right or/and the best, we need to have a *previous* understanding of the four notions involved, more sophisticated than just cutting the pears in halves. Our roadmap to the final solution, like all good tragedies, as five steepes:

1. Cut the two pears in halves: *a priori* / *a posteriori* - *analytic* / *synthetic*
2. See all the ways to put the two dichotomies in a square within a hexagon
3. Sustainable understanding of the four notions involved
4. Based on (3), choice of the right figure among those settled in (2)
5. Interpretation and justification of the result

Fig. 11 – Road map to the final solution

The first two steps have already been performed: the first step by Kant, the second by Fig. 10. These two steps are *a priori*. The third one is rather empirical. It can be performed by reading the works of Kant (cf. [24], [25], [26]) during a walking pilgrimage to Königsberg (nowadays better known as “Kaliningrad”). And if there are still doubts, one can read them again in reverse order on a walking pilgrimage back home.



Fig. 12 – Walking pilgrimage to Königsberg

There are other methodologies for business people who do not have plenty of time. One is to read second sources: good papers or/and encyclopedias (see e.g.[18], [22], [27], [28], [31]) , another one is to appeal to artificial intelligence, for example the friendly *Chat Gpt*.

This can be very useful if we use A.I. in an intelligent way. A.I. is not the final stage of intelligence. Chat Gpt makes a good synthesis of what Kant and commentators have said on the topic. It is in the Equatorial zone (cf. Fig. 7) : not too much descriptive, not too much normative, like a good encyclopedia. It makes perfectly sense if our objective is no to understand what Kant himself have exactly thought, but to develop an autonomous theory based on his ideas. And we do not intend to use Chat Gpt in a purely passive way but within a five-step road map (cf. Fig. 11).

Here is how Chat Gpt is decrypting the situation (cf. [20] and [21]):

The distinction between **a priori** and **a posteriori** is a fundamental one in epistemology (the theory of knowledge), especially prominent in the philosophy of Immanuel Kant.

◆ **A priori knowledge**

- **Definition:** Knowledge that is independent of experience.
- **How it is known:** It can be known through reason alone, without needing to refer to specific sensory experience.
- **Examples:**
 - "All bachelors are unmarried." (True by definition)
 - " $7 + 5 = 12$ "
 - "If something is red, then it is colored."

These statements are knowable **a priori** because they can be justified purely by thinking or by understanding the concepts involved.

◆ **A posteriori knowledge**

- **Definition:** Knowledge that depends on experience or empirical evidence.
- **How it is known:** It can only be justified or confirmed through observation, perception, or scientific investigation.
- **Examples:**
 - "It is raining outside."
 - "Water boils at 100°C at sea level."
 - "Brazil is in South America."

These are **a posteriori** because you need to look at the world to verify them.

🔄 **Summary of the Difference**

Feature	A Priori	A Posteriori
Depends on experience?	No	Yes
Based on reason alone?	Yes	No
Example	"All triangles have 3 sides"	"The Eiffel Tower is in Paris"
Typical domain	Logic, mathematics	Natural sciences, geography

Fig. 13– Chat Gpt about *a priori* / *a posteriori*

The distinction between **analytic** and **synthetic** statements or judgments is a classic one in philosophy, particularly emphasized by **Immanuel Kant**. Here's a clear and structured explanation:

◆ **Analytic Statements**

- **Definition:** Statements that are **true by virtue of their meaning alone**.
- **No new information is added** in the predicate that isn't already contained in the subject.
- **Truth is based on logic and definitions**, not on empirical observation.

Examples:

- "All bachelors are unmarried."
- "A triangle has three sides."
- "If something is red, it has a color."

These are **true by definition**. You don't need to check the world to confirm them — just understand the meanings of the words.

◆ **Synthetic Statements**

- **Definition:** Statements that are **true or false based on how the world actually is**.
- The predicate adds **new information** not contained in the subject.
- **Truth is based on experience or empirical evidence**.

Examples:

- "The cat is on the mat."
- "Water boils at 100°C."
- "All bachelors are lonely."

These require **observation or investigation** to verify.

◆ **Kant's Contribution**

Kant combined this with another distinction: **a priori** (known independently of experience) and **a posteriori** (known through experience). He famously claimed that:

Mathematics consists of **synthetic a priori judgments**.

For instance:

- " $7 + 5 = 12$ " is not just a matter of analyzing the concepts "7," "5," and "+" — the concept "12" is **not contained** in those — yet it's **known independently of experience**.

◆ **Summary Table**

Type	Truth Basis	Example	Needs Experience?
Analytic	Meaning/definitions	"All bachelors are unmarried."	✗ No
Synthetic	Facts about the world	"The sky is blue."	✓ Yes

Fig. 14– Chat Gpt about *analytic / synthetic*

The dichotomy *a priori* / *a posteriori* is related to experience. A knowledge is *a posteriori* if it depends on external experience, it is *a priori*, if it is the pure product of our mind. This latter dichotomy can be formulated using the term “empirical” together with classical negation: *empirical* / *non-empirical*.

The dichotomy *analytic* / *synthetic* is about what was called at the time of Kant “judgements”, that we can call nowadays simply “propositions”. A typical case of analytic proposition is related to Kant’s personal life: “All bachelors are unmarried”. The truth of this proposition is based on the inner meaning of the concepts involved in it. But the knowledge that Kant was a bachelor is *a posteriori*.

It is important to note that these two dichotomies are not about the same kinds of objects. On the one hand, we have knowledge, on the other hand propositions. It is like two complementary versions of the same view, that can be represented in an analogical way:

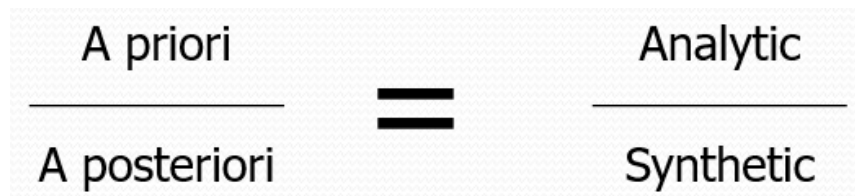


Fig. 15 -

In some sense, *a priori* speaking, this kind of analogy can be made about any dichotomy. But thinking of the meanings of the two notions involved, we have a thickness greater than the fraction line. Also, the equality sign is just an extreme case. We can reformulate this analogy with the following picture:



Fig. 16 -

What we can say is that *analytic* and *a posteriori* are disjoint or, to speak in a less extensional way, exclusive. For this reason, neither *analytic* implies *a posteriori*, nor *a posteriori* implies *analytic*. Even if the statement “All bachelors are unmarried” is true, this does not mean we know that bachelors are unhappy and vice versa. Thus, we can eliminate the two bottom hexagons of Fig. 10.

We are left with the two top hexagons:

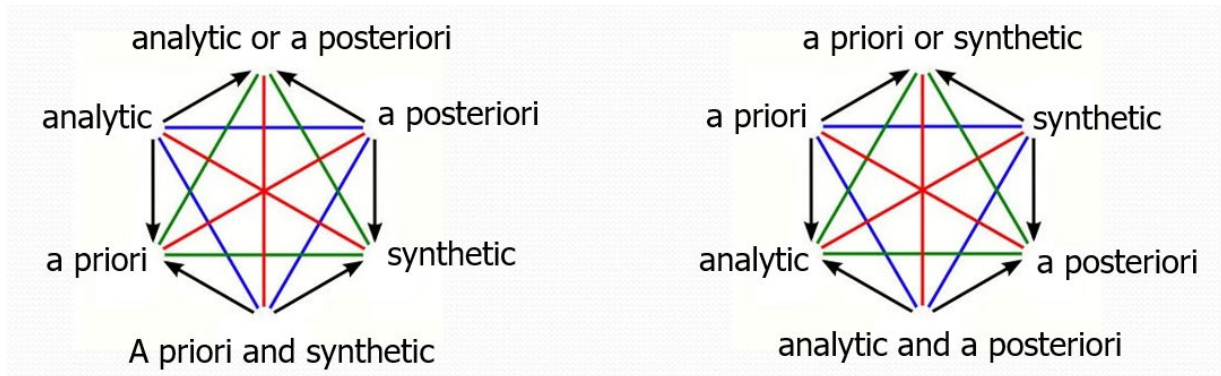


Fig. 17 - The two valuable hexagons to put together the dichotomies *analytic/synthetic- a priori/a posteriori*

According to the right hexagon, everything which is *a priori* is *analytic*. It gives a sterile picture of pure reason, that was rejected by Kant who promoted the fruitful combination of *a prioricity* and *synthesis*, mainly through the mathematical example of $7 + 5 = 12$.

Kant's philosophical views on mathematics based on *a priori* intuitions of space for geometry and time for mathematics have been heavily criticized. But we can support the combination of *a prioricity* and *synthesis* without sticking too much to Kant's philosophy of mathematics.

The discovery of an important mathematical truth is not something immediate and obvious, it requires time and mental activities to make such a discovery, which may be a pure activity of reason leading to a resulting proposition that can be considered as synthetic, like the fact that there is an infinite quantity of prime numbers, to take a less trivial example than Kant's one.

The same can be said of the irrationality of square root of two to take another example of a mathematical result of ancient Greece. The proof of this theorem by reduction to the absurd is purely abstract and *a priori*, nevertheless it is not analytic, because $\sqrt{2}$ is not rational by definition, the irrationality of this number is not contained in it in the way that bachelors are unmarried, that a hexagon has six sides, or that God exists (according to Anselm of Canterbury, not according to Kant or Königsberg! Cf. [23], [33]).

5. The meaning of the two Kantian pairs according to the hexagon

So at the end we are left alone with the following hexagon:

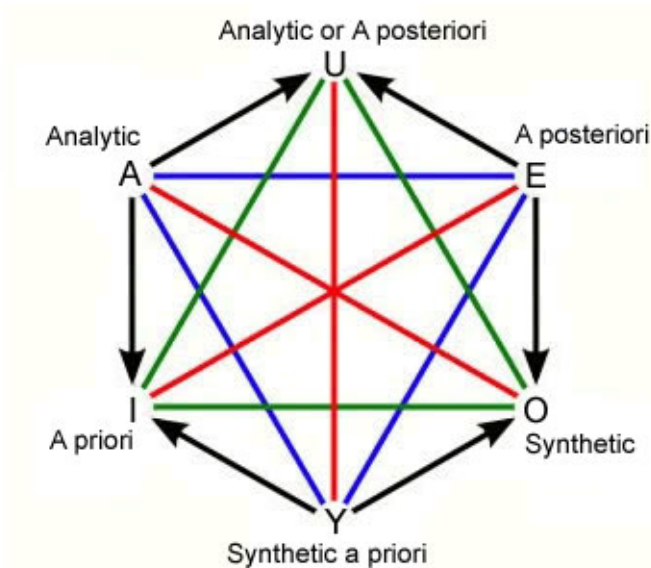


Fig. 18 – *Synthetic a priori* in a hexagon of opposition

This hexagon gives us a better understanding of both famous Kantian dichotomies by relating them. Among the four hexagons of Fig. 10 that describe all the possible relations between these two dichotomies within the framework of the theory of opposition, it is the only one supporting the central notion of *synthetic a priori*, as Kant did. This is the closest to Kant's ideas, but it does not mean that it is absolute truth or/and that it perfectly reflects Kant's ideas. The six notions of these hexagons can be interpreted in different ways, they have different *models*, in the sense of model theory (see [13] and [29].)

According to Kant, the two dichotomies are on different plans: one (*analytic/synthetic*) on the plan of propositions, the other (*a priori/a posteriori*) on the plan of knowledge, but he himself mixed the two promoting the notion of *synthetic a priori*. Does *synthetic a priori* apply to proposition or knowledge? What we can say is that a mathematical proposition like $5+7=12$ is a *synthetic proposition* based on *a priori knowledge*. The same applies even more strongly to the proposition according to which $\sqrt{2}$ is not a rational number.

If we define/present *a priori knowledge* as a knowledge which is independent of *experience*, an ambiguity should be clarified. One can interpret here "experience" as something related to the external world not the pure product of reason. This does not mean that knowledge of pure reason is always something obvious and trivial, it can be the product of an investigation, whether we call it experience or not.

Acknowledgements

This hexagon was already presented in a published paper as early as 2012 (see [2]) but with few explanations. It was presented again more recently (2023) in a joint paper with Srećko Kovač and Jens Lemanski in German about Kant's logic (cf. [12]) with more details, but still quite briefly and not in full flesh.

The ideas of this paper were presented in a talk at the 7th edition of the *World Congress on the Square of Opposition* which took place at the Catholic University of Leuven (KUL) in September 2022. I would like to thank all the people with whom I have discussed this topic along the years in particular, but not only, Srećko Kovač.

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