Review of Geraldine Brady, *From Peirce to Skolem – A neglected chapter in the history of logic*, Studies in the history and philosophy of mathematics, vol.4, Amsterdam, North-Holland / Elsevier, 2000. xi + 468p.

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This book sheds new light on the birth and development of modern logic, showing the important role of Charles Sanders Peirce.

Jean van Heijenoort can be considered as the first historian of modern logic. In 1967 he published a book entitled *From Frege to Gödel*, a collection of some crucial papers by famous logicians accompanied by erudite introductions and comments. However, as we know now, Heijenoort's view is highly distorted, particularly the way he presents the role of Gottlob Frege.

G.H.Moore in his review of the second edition of *From Frege to Gödel* pointed out that Frege is almost never quoted by other authors of papers collected in this book which is supposed to depict the road leading from Frege to Gödel. Another drawback in Heijenoort's book is the absence of papers from the Polish school, particularly by Alfred Tarski, the leading logician with Kurt Gödel of the XXth century.

Recently I.Grattan-Guinness has written a book entitled *The search for mathematical roots 1870-1940* which in many senses gives a new vision of the first period of the history of modern logic. In the subtitle of the book he uses the expression "from Carnot through Russell to Gödel", correcting the title of Heijenoort's book by giving a new perspective. A good point of Grattan-Guinness's book is that he puts Frege in his rightful place and he talks quite a lot about Ernst Schröder who is completely ignored by Heijenoort. But his book also has the drawback of not giving the right account of the Polish school.

Geraldine Brady's book is quite different from Heijenoort's and Grattan-Guinness' books. It is not a collection of papers like Heijenoort's book and it is not a general description talking about everybody and everything like Grattan-Guinness'one. It is focused on a precise description of a limited but central part of the beginning of modern logic. But at the same time Brady is someone with a solid background and is able to make interesting connections, for example, between the work of Peirce and topos theory (p.20).

In her introduction, Brady talks about two papers which are considered fundamental in the history of modern logic: Leopold Löwenheim's paper of 1915 showing that if a statement of first-order logic has an infinite model, it also has a countable model, and Toralf Skolem's paper of 1923 giving a new proof of Löwenheim’s theorem which is very similar to Gödel’s 1929 completeness proof as indicated by Heijenoort and Hao Wang.

These two papers have been translated into English and presented in the book by Heijenoort who always considered them as fundamental. But what is the road from Frege to Löwenheim-Skolem? Brady, after a detail study, claims: “We have been unable to detect any direct influence of Frege, Russell or Hilbert on the development of Löwenheim and Skolem’s seminal work, contrary to the commonly held perception” (p. 2).

On the other hand, she says that “Löwenheim’s and Skolem’s work on what is now known as the downward Löwenheim-Skolem theorem developed directly from Schröder’s *Algebra of Logik*, which was itself an avowed elaboration of the work of the American logician Charles S.Peirce and his student O. H. Mitchell.” (p. 2). Her book traces this development and it is therefore rightly called *From Peirce to Skolem*.

More than half the book consists of appendices of about two hundred pages of translation of parts of Schröder’s *Vorlesung über die Algebra der Logic* followed by fifteen pages of excerpts from the thesis of Norbert Wiener (introduction and last chapter), the founder of cybernetics, a thesis which is a “careful examination of Schröder’s algebra of relatives vis-à-vis Russell’s treatment of relatives in *Principia Mathematica.*” (p. 429).

The other half of the book, the first two hundred pages, consists of Peirce’s work and that of his student, Mitchell, followed by a description of Schröder’s calculus of relatives and then short presentations of Löwenheim’s and Skolem’s papers in this light.

In this book, Brady presents material which is very important to understand the development of quantification, first-order logic, and the theory of models. Tarski was the first to use the expression “theory of models” in the 1950s and turned this field into the main line of logical research during the thirty years he was at Berkeley with his team. As Solomon Feferman writes: “Tarski did not create this field, but as Robert Vaught later wrote in a survey of it for the occasion of Tarski’s seventieth birthday celebration, his influence was decisive” (2004, p. 280). In several places in her book Brady mentions Tarski and some influence on his work by Peirce, Schröder, Löwenheim and Skolem. Tarski’s model theory, however, is a mixture of the algebraic school with the proof-theoretical and syntactical line of Frege-Russell-Hilbert. Wilfrid Hodges (1985-86) in an interesting paper explains why the famous notion of truth in a structure was given by Tarski only in the 1950s in his papers on model theory, this is connected to this problematic mixture.

But in his last work, his book with Givant (1987), Tarski comes back to the algebraic and “relational” tradition, which was the first way that modern logic was introduced in Poland (see Wolenski, 1989, p. 82). In the Preface of their book, they write: “The mathematics of the present book is rooted in the calculus of relations (or the calculus of relatives, as it is sometimes called) that originated in the work of A. De Morgan, C. S. Peirce and E. Schröder during the second half of the nineteenth century” (1987, p. xv).

The present book by Brady is not intended to be a general presentation of Peirce’s contribution to logic or a general description of Peirce’s influence on the development of logic. For example, nothing is said about Peirce’s contribution to many-valued logic (but Brady says a few words on truth-tables, pp.125-126), or on the influence of Peirce on the Russian school (on this topic, see Bazhanov, 1992). No such book, giving a global perspective and detailed account of Peirce work’s in logic, has been written yet - this is surely not an easy task. But ten years ago a collection of papers was published dedicated to the work of Peirce in logic: *Studies in the logic of Charles Sanders Peirce* which includes a 20 page paper by Brady entitled *From the Algebra of Relations to the Logic of Quantifiers*, which can be seen as a summary of the book under review.

Nevertheless this book gives an interesting presentation of many aspects of Peirce’s work in logic, for example, the relation between Peirce’s work and George Boole’s work, and also the influence on the work of Peirce of his father, the mathematician, Benjamin Peirce. Here are Brady’s comments about “Description of a notation for the logic of relatives”, the first work of Peirce she presents in her book: “The notational system practically fell into Peirce’s lap entire by analogy with his father’s work in linear algebra. An individual term is like a coordinate, an absolute term is like a vector, and a relative term is like a matrix or linear transformation. This sparked a whole area of logic, matrix logic” (p.48).

Brady’s book is very important in making the work of Peirce in logic better known. His work is still largely under evaluated despite the fact that Peirce has become a legendary personage. In their famous book, *The development of logic*, W.Kneale and M.Kneale write: “unfortunately Peirce was like Leibniz, not only in his originality as a logician, but also in his constitutional inability to finish the many projects he conceived” (1962, p.427). Kneale and Kneale’s book, like Heijenoort’s book, is largely Fregean with four chapters bearing the name of Frege. They say few things about Peirce’s work and its influence on the development of modern logic. But even if Peirce, as they say, was not able to finish his projects, his work was pursued by others, especially by Schröder as Brady shows in her book, which led to the first important result in modern logic: the Löwenheim-Skolem theorem.

As Brady says: “It is not always clear to what extent Peirce’s work influenced later developments in logic and to what extent it simply anticipated them” (p. 11). Peirce had many important ideas that later on became famous subjects of studies, not only in logic, but also in philosophy and semiotic. It is true that it is not easy to see a causal link between Peirce’s work and later works by other logicians. For example, in the case of many-valued logic, Emil Post, Jan Lukasiewicz, or Paul Bernays didn’t take the idea from him. Brady also says that Peirce anticipated natural deduction, but notes that there probably was not a direct influence by Peirce on Gerhard Gentzen and Dag Prawitz.

This absence of a causal link is an interesting and important point that historians should meditate on. History is most of the time conceived according to a kind of causality principle: what caused what, who influenced whom, which idea generated another idea. But one may have another perspective, pointing out ideas that were floating in the air. Obviously in the history of modern logic one can see that different people had the same idea at the same time, independently. If we want to apply a causal approach to the history of modern logic, many things appear as mysterious, and maybe this part of the history of logic is a good counterexample of causal perspective.

In her book, Brady gives many examples of such mysteries, for example the rise of David Hilbert’s notation as the main notation for logic. Some people, like Bertrand Russell, say that one of the reasons for the weak influence of Schröder’s work was his cumbersome notation, but Brady comments: “However, notational complexity alone does not necessarily explain his neglect. Frege’s conceptual notation and his *Grundgesetze* are often equally unreadable, as is Whitehead and Russell’s *Principia Matematica*, especially volume 3. The current notation for first-order logic comes from none of them; it arrives full blown in Hilbert’s 1917 lectures, without any reference to anyone.” Let us recall that Gentzen, from the Hilbertian school, introduced later on the symbol for the universal quantifier, $\forall$, finalizing this dominant way of writing logic.

Recall that half the book is a translation of parts of Schröder’s *Vorlesung über die Algebra der Logic* connected with the topic under discussion. Brady did this because Schröder’s book is still not translated into English. We can hope that one day the full book will be translated (but then a further edition of Brady’s should perhaps be reduced to two hundred pages), which will be very useful for a better appraisal of Schröder in the history of modern logic (and at same time of his mentor, Charles S. Peirce). We remember also that another monumental book of the first period of modern logic has still not be translated into English, the book by Hilbert and Bernays, *Grundlagen der mathematik*, which has been translated recently into French by F.Gaillard and M.Guillaume.

The last appendix of Brady's book consists of some excerpts from Norbert Wiener's Ph.D.'s thesis which is not very well known and not yet published. Maybe its presentation in Brady's book will encourage someone to prepare a publication of this interesting work.

Anyway, Brady's book is a very important contribution to the history of modern logic, useful for anyone who wants to better know the subject and understand Peirce's role. The book is very well written and organized, with good appendices, bibliography and index.

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