Are there any intelligent forms of opposition?
Jean-Yves Beziau explains how we can better think using the square of opposition

In the beginning was dichotomy

The expression "the square of opposition" can be associated to something like Tienanmen square: opposition in a square. For many, opposition means political opposition, which can vary from animated discussions to wars. But opposition can be seen less dramatically as a basic feature of our thought, a useful tool which can drive us to a better understanding of reality, if not a gate for heavenly peace.

Thinking is based on establishing identifications and differences. The concept of cat gathers together a series of many different beings which all have something in common and are different from cars, entities which are put together in a different bag of our mind. There are many different differences: cats are different from tigers, eagles, tomatoes and computers. Differences can range from similarity to opposition. And opposition can range from light opposition to diametrical opposition.

It is attributed to the school of Pythagoras a table of basic oppositions which were used to explain everything:

TABLE OF OPPOSITES	
One	Many
Rest	Motion
Right	Left
Male	Female
Odd	Even
Finite	Infinite
Straight	Curved
Light	Darkness
Good	Evil
Happy	Sad
Open	Close
In	Out

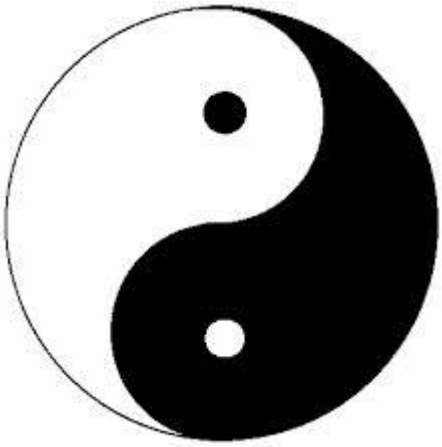
Plato much influenced by Pythagoras went further on promoting dichotomy as a methodology for thinking splitting everything in two parts, as exemplified when chasing the sophist in the dialogue of the same name.

One may find dichotomy too strong, an exaggeration. Is there nothing beyond comedy and tragedy, music and noise, good and evil, chance and determinism? Should we be crazy or bore, believer or atheist, democrat or republican? Are there no other choices? Can we not be neither or both? Loving

cats and dogs, or just birds. And even without considering creatures like shemales, one may argue that a male as a feminine part and vice versa.

Oriental opposition vs. occidental opposition

Maybe Taoist philosophy is a subtler way of thinking. Does not the Yin Yang symbol express the idea that there is Yin in Yang and Yang in Yin? Is not the distinction between Yin and Yang a more subtle opposition than Pythagorean dichotomies? The great Danish physicist Niels Bohr, long before it was fashionable, liked to wear the Tao symbol. For him it was the mirror of the paradoxical duality wave/particle as it appears in modern physics.

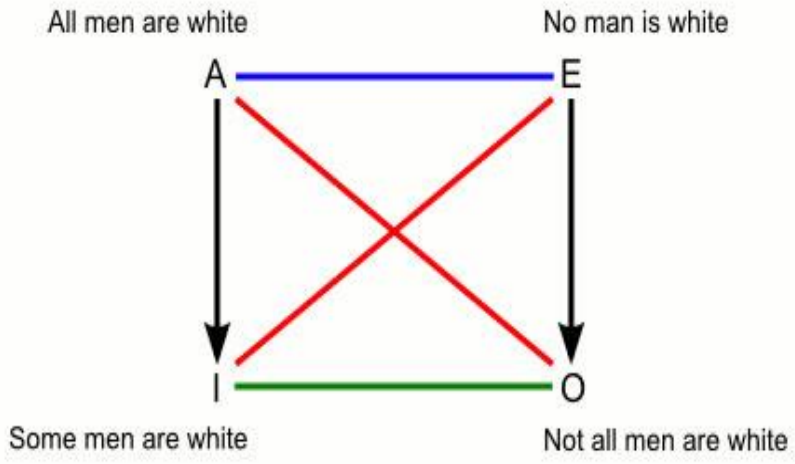


But making a strong opposition between the Occidental and the Oriental, confronting Pythagoreanism to Taoism, is also to follow the wrong road of dichotomy leading us back to cheese or cake, hell or paradise. The ancient Greeks were themselves able to go beyond dichotomy, not only in a mythological way with the androgynous creatures of Plato's Symposium but also in a systematic and rational way. That's the story of the square of opposition. This square is often associated to Aristotle; he himself didn't draw any square but promoted an idea which is the first step towards it, the idea of *contrariety*, an opposition weaker than contradiction. A square and a circle are contrary because something cannot be at the time square and circle, but they are not contradictory because something can be neither square, nor circle, triangular for example.

However even if this distinction was operated by Aristotle twenty five centuries ago, it is still not very clear in human thought, as manifested by our way of speaking. "Contrariety" is a technical word not used in ordinary language and "contrary" is generally used meaning the same as contradictory. For us the opposition between circle and square is the same as between odd and even. But if a number cannot both be odd and even it is also necessarily odd or even: you can divide it in two or not.

The square of three oppositions

And there is a third notion of opposition, even less clear for us; it is the dual of contrariety, called *subcontrariety*. The easiest way to explain it is to draw the square of opposition:

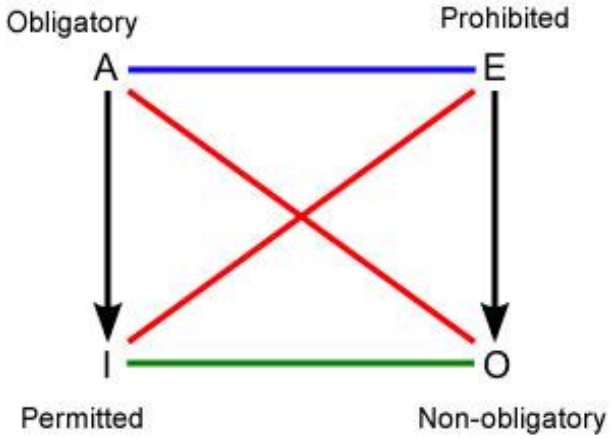


This square describes the relations between four propositions. In blue we have the relation of contrariety, in red of contradiction, in green of subcontrariety. Two propositions are said to be *contrary* iff they cannot be true together but can be false together, *contradictory* iff they cannot be true together and false together, *subcontrary* iff they cannot be false together but can be true together. These are the three notions of opposition and in black we have what is now called implication, but what was traditionally called *subalternation*.

This square was not drawn by Aristotle in particular because he didn't conceived subcontrariety, this was done later on by Apuleius (125-180) and in a final stage by Boethius (480-524). The square of opposition is nevertheless associated with Aristotle, not only because he conceived the notion of contrariety, a fundamental move towards the square, but also because the square was developed and used to represent Aristotle's theory of propositions, a classification of propositions in four categories. This is reflected in the names given for the corners: A for *Affirmatio universalis*, E for *nEgatio universalis* , I for *affIrmatio particularis*, O for *negatiO particularis*

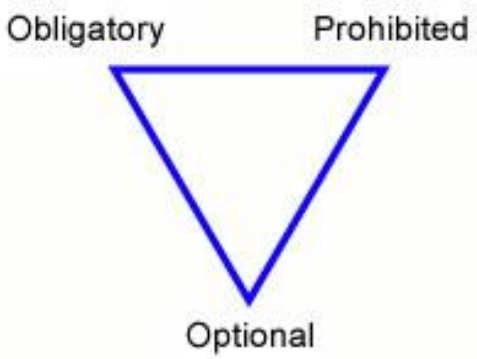
From the square of obligation to the optional triangle

The articulation of oppositions presented in the square is however not limited to such theory of propositions or even to propositions. It applies to any notions that can be expressed in a way or another. Here is for example the square of obligation (generally called the *deontic square*):



The top part of this square tells us that something cannot be at the same time obligatory or prohibited. This makes sense: it cannot be at the same obligatory to smoke and prohibited to smoke. But someone may argue that prohibition is a kind of obligation. Yes, but it is a negative obligation. We can express the prohibition of smoking as the obligation of not smoking.

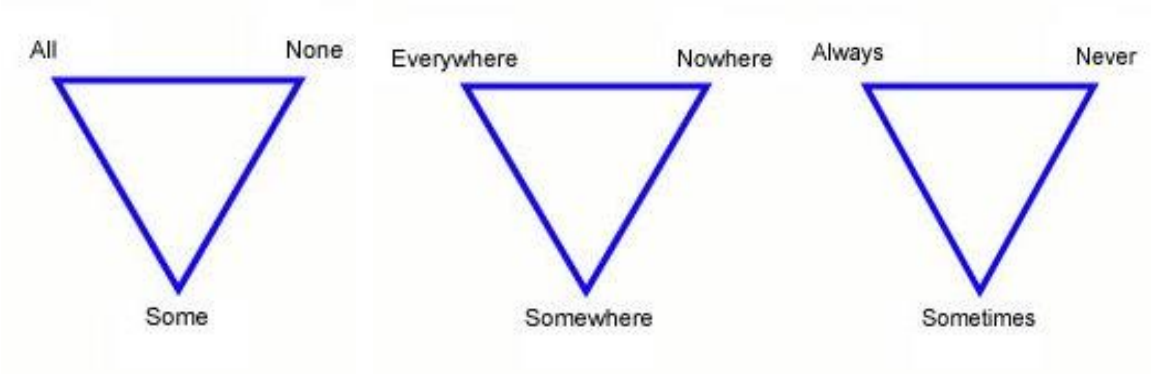
Now if something is not prohibited, it is permitted, and if it is not permitted, it is prohibited, so it makes sense to say that permission and obligation are contradictory. But is something permitted obligatory? The deontic square doesn't say that a permission is necessarily an obligation but it does not rule out the possibility of a permission to be an obligation. It makes sense to say that something which is obligatory is permitted, but when I am saying that something is permitted generally I don't want to say that it is obligatory. To avoid confusion it would be useful to have a clear-cut name for that. A good name is "optional". And we have then the following triangle.



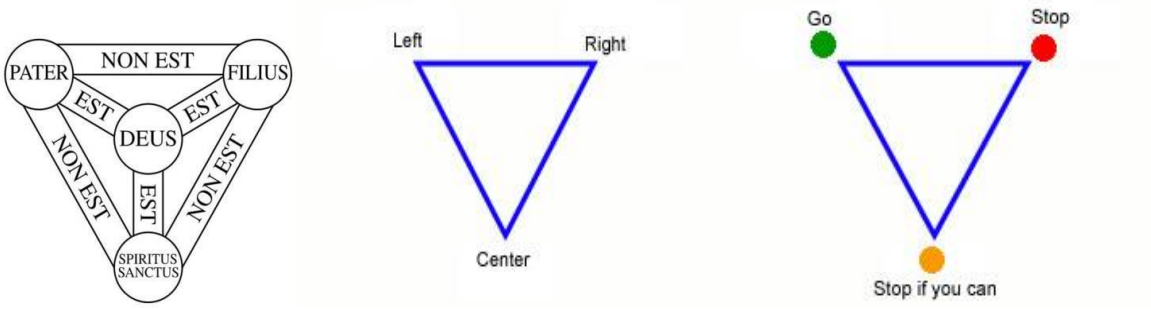
This is the optional triangle or *deontic triangle of contrariety*. The relations between each vertex are contraries: something cannot both be obligatory and prohibited, obligatory and optional, optional and prohibited, but something can be neither obligatory nor prohibited – it can be optional, neither obligatory nor optional – it can be prohibited, neither optional nor prohibited – it can be obligatory.

Everything is triangular?

Even if the word "contrariety" is not present in our language, our thought is based on a lot of triangles of contrariety:



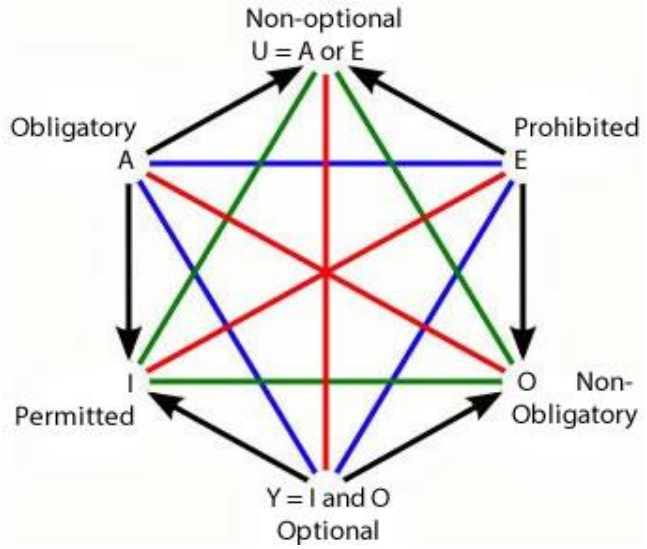
We are surrounded by triangles of contrariety:



So one may think that contrary trichotomy is the basis of everything. But shall we forget the square and the other oppositions it comprehends: contradiction and subcontrariety?

From the contrary triangle to the hexagon of opposition

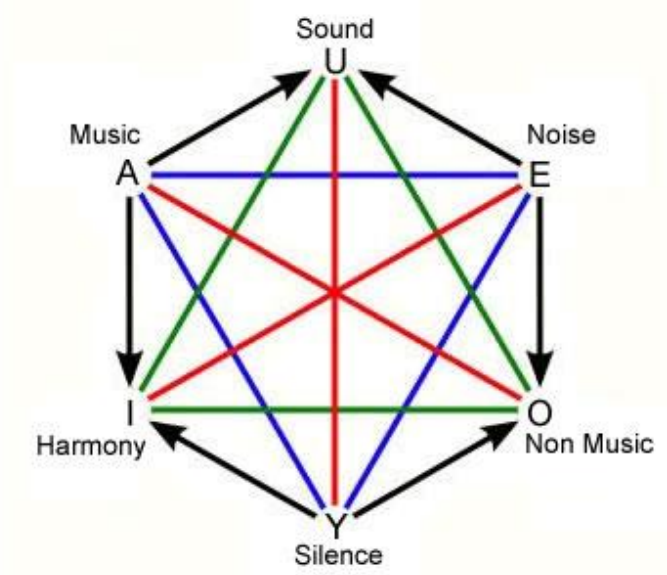
Robert Blanché (1898-1975) has shown that in fact it is possible to reconstruct the square of opposition with a hexagon built by assembling the triangle of contrariety with a triangle of subcontrariety. Let us have a look at the deontic hexagon:



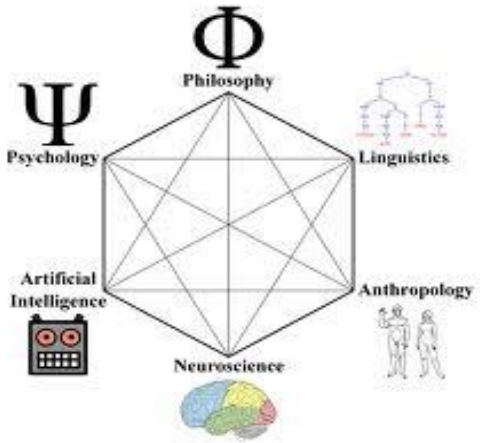
We find here both “permitted” and “optional”. Something which is optional, standing at the Y vertex, is something which is permitted and non-obligatory. The top vertex is the contradictory of optional and at the same time it is defined as something which is obligatory or prohibited, perfectly describing the common feature of the two notions of “obligation” and “prohibition” (positive or negative obligation). Whether it is obligatory to smile or prohibited to smile, you have no options.

Magic hexagons and squares

Hexagons can be constructed to explain the interrelations of many concepts. For example we can draw the following hexagon of music:

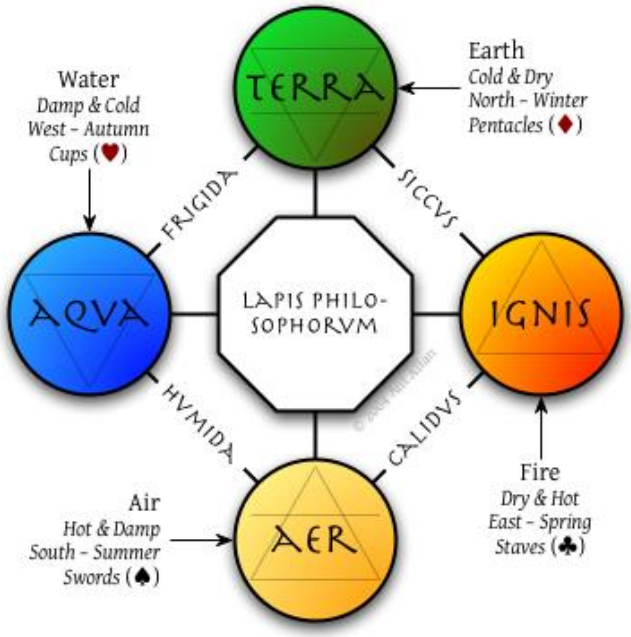


Now let us have a look at the following hexagon which is used to describe cognitive science:

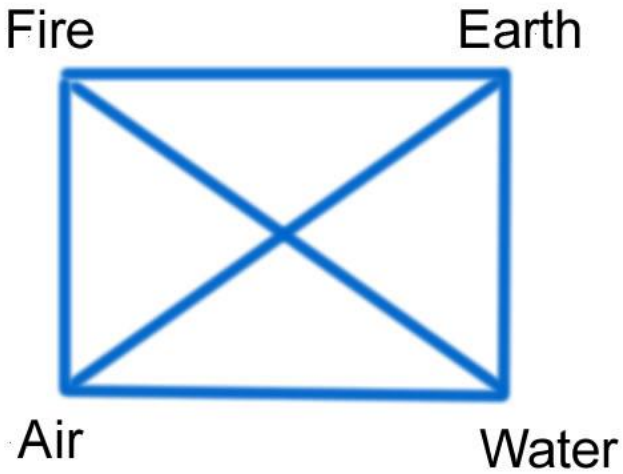


Such hexagon has nothing in common with the hexagon of opposition, apart the fact of being a hexagon. It is a superficial similarity. The lines inside the hexagon just express that all the six fields are related to each other, but the detail of these relations is not explained.

At the level of the square we can find similar things, like the so-called, *elemental square*, or square of the four elements: earth, wind, air and fire:



Such square is not a square of opposition with three oppositions. At best it can be considered as a contrary square, where the relation between each pair of elements is contrariety:



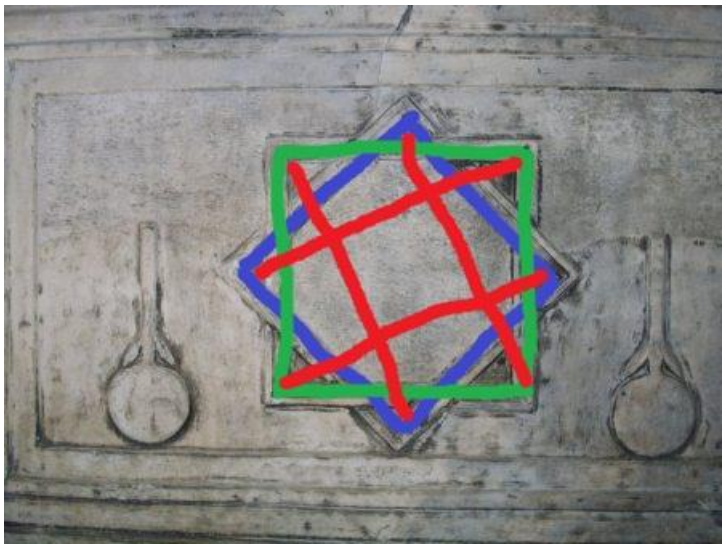
What is the value of such contrary quatritomy? That's not clear. It is a way to reduce reality to some basic elements. But why four? The Chinese have an additional elements, woods. This can be represented by a blue pentagon of contrariety.

A blue pentagon was also used in Geneva in 1939 as a symbol for the League of Nations. The number five here represents the five continents and the five races of mankind.

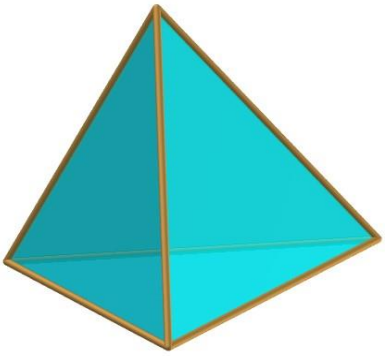
This is a nice symbol but it was not powerful enough to stop the war and the second world war was the end of the league of nations.

Polytomy and oppositions in the third dimension

Such geometric forms may have an emotional impact but they are not representing very sophisticated mental constructions. What is interesting is that we can generalize the construction of the hexagon to any polytomy. Instead of starting with a triangle of contrariety, we can start with a square of contrariety and construct a decagon built with a square of subcontariety. We can then transform the double square, basic symbol of Islamic religion, into a meaningful decagon of opposition. Here is how we can put the three oppositions in the double square than can be found in the famous mosque Hagia Sofia in Istanbul:



This oppositional structure can be represented in a better way, following an idea of Alessio Moretti in his PhD *The Geometry of Logical Opposition* (2009). Instead of considering a square of contrariety it is better to consider the following polyhedron:



There are also four vertices but the difference with a square is that the distance between all vertices is the same, which makes sense if we are considering four notions which are equally contrary to each other. A polygon having such a property is called a *simplex*. We can put two simplexes together, a simplex of contrariety and a simplex of subcontrariety, tying them by contradictory axes. We have then a bi-simplex which, in the theory of polyhedra, is called a *tetrahedron*. In Islamic culture people like to use such a polyhedron as a lamp. The tetrahedron can also be seen as a light for our mind, organizing our thought in a complex but clear symmetrical structure.

Squaring the world

In 2007 was organized in Montreux. Switzerland the first world congress on the square of opposition gathering logicians, philosophers, semioticians, linguistics, mathematicians, psychologists and artists. Then a second edition of this event was organized in 2010 at the University Pasquale Paoli in Corté, Corsica, a third edition in 2012 at the American University of Beirut in Lebanon and a fourth edition in the Vatican at the Pontifical Lateran University.

Many important researchers have taken part to these events: Terence Parsons, Jan Wolenski, Pascal Engel, Pieter Seuren, Dale Jacquette, Pierre Cartier, Damian Niwinski, Dany Jaspers, Wolfgang Lenzen, Peter Schröder Heister, Hans Smessaert, Laurence Horn, Stephen Read, Claudio Pizzi, John Woods, Lorenzo Magnani, etc. Several publications have followed these events. You can find details on the page http://www.square-of-opposition.org

November, 11-15, 2016 the fifth edition of the square congress will happen in Easter Island.

Jean-Yves Beziau is professor of philosophy at the Federal University of Rio de Janeiro and editor-in-chief of the journal Logica Universalis. His webpage is http://www.jyb-logic.org.