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WHAT IS SYMMETRY?

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Abstract: *We ask the question “What is symmetry?” in the perspective of developing a general methodology to answer, for any notion X, the famous question promoted by Socrates and Plato “What is X?”. First, we present a theory of meaning where “notion” is the top key surrounding a triangle made of word-idea-thing. After discussing the meaning of symmetry and showing some examples, in particular point out that all Lindenbaum-Tarski logical notions are symmetrical, we present a pyramid of symmetry. In a second part, we use a structural methodology based on associations and after exhibiting some smoggy word clouds for symmetry, we draw a pure word cloud for symmetry. Thirdly we apply the theory of oppositions designing a symmetry hexagon, establishing the relations and distinctions between symmetry, antisymmetry, asymmetry, dissymmetry, giving examples including human body and storm.*

Keywords: *Pyramid of Symmetry, Word Cloud of Symmetry, Hexagon of Symmetry*

AN UMBRELLA NOTION AS A PLATONIC GUINEA PIG

Symmetry is an umbrella notion covering a broad spectrum of ideas ranging from a mathematical notion (close to general abstract nonsense) to a variety of concretes cases, through some geometrical lanterns than can illuminate our brain. We inquire here how to catch such a notion through the famous Socratico-Platonic question *What is X?* We present three ways to answer the question *What is symmetry?* than can also be applied to any notion: the pyramid of meaning, the word cloud and the theory of oppositions, using symmetry as a guinea pig for elaborating a general philosophical methodology.

1. PYRAMID OF SYMMETRY

When asking “What is symmetry?”, towards *what* is the question heading? Obviously not only to the word “symmetry”, but also to its meaning. But what is the meaning? Is it a reality or a conception? To speak of the reality of symmetry seems a bit weird because it is rather a concept. It would look more natural if we were asking the question “What is the sun?”.

But to face the question “What is X?” whatever “X” is, it is good to have a general methodology. A pivotal aspect of our methodology is to consider X as a *notion*, viewed as the top of a pyramid of meaning whose basis is a triangle with *word-idea-thing* as corners, which can be presented in a more phenomenological way as the triangle *language-thought-reality*. From this perspective, the question “What is X?” is heading towards the notion X considered as encompassing the three aspects of such a triangle. We have to look at the three corners, but not separately. Here is the general setting of the pyramid of meaning:



Fig. 1 The Pyramid of Meaning

If we consider the word “symmetry”, one important thing to note is that it a substantive, but there is an adjective associated to it “symmetrical”. It is not the case of every substantive, for example there is no adjective associated with “chair”. Also, there is a verb associated to it: “to symmetrize”, although it is more artificial that “raining” associated to “rain”, case where we also have the adverb “rainy”. All this means that

“symmetry” appeared rather as a quality than as an object, although it may be considered as an abstract reality, a kind of relation.

A standard definition of symmetry runs indeed as follows:

◆ AI Overview

Symmetry, as defined by Oxford Languages, is the quality of being made up of exactly similar parts facing each other or around an axis. This means an object or shape can be divided into two or more identical parts, either by a line, point, or axis, where each part mirrors the others. [↗](#)

In simpler terms, symmetry implies a balanced and harmonious proportion between the parts of an object. This concept is widely applicable, appearing in nature, art, architecture, and mathematics. [↗](#)

Fig. 2 Characterization of symmetry by Google Gemini

When investigating a notion, it is always good to have a look at a dictionary, although this should not be taken as the final solution, if any. A dictionary heads towards the reality mainly through an idea.

What about the next step: how to reach reality? We may face the reality of the sun having a look at it. In the case of a notion, which is rather conceptual like symmetry, it is more difficult. But when it is qualitative, we may look at objects having this property. The risk it to be lost in a chaotic multiplicity, but this is true also for notions like stone, beauty or number.

To find a way out such a diversity is to guide ourselves with some *typical examples* which can be natural, artificial, or symbolic.



Fig. 3 Typical Examples of Symmetry

These are “concrete” examples of symmetry. Opposed to that, we have the definition of the relation of symmetry in mathematics, which is also rather a property: a *binary relation*

is symmetrical when aRb iff bRa . There is a huge gap between concrete symmetrical objects or phenomena and such an abstract relation which cannot really be considered as a satisfactory modelization, in the sense that models of this relation, in the sense of model theory, correspond to the class of concrete symmetrical objects.

However, this abstract formulation can be used to develop an understanding of what is different from symmetry as we will explain in section three. Moreover, an important point of this abstract relation is that it is part of the definition of the notion of identity defined as a *congruence relation*, a specific case of *equivalence relation*, a relation which is reflexive, symmetrical and transitive. This is interesting because symmetry itself is based on identity, or a weak form of it, similarity.

But we have to be careful because symmetry is not restricted to identity, it is in fact a common property of all the four Tarski-Lindenbaum's logical notions, they are all symmetric, as we can see in the following example with two elements (that your imagination can easily extend to any number of elements):

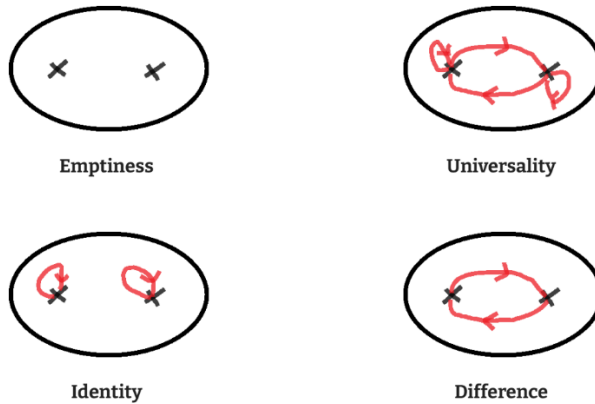


Fig. 4 All Four Logical Notions are Symmetrical

This does not mean that all symmetric relations correspond to one of the four logical notions. For example, the following one is corresponding to none of them:

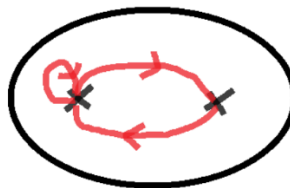


Fig. 5 A Symmetrical Non-Logical Relation

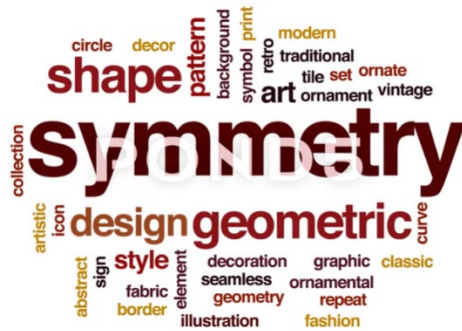


Fig.7 Three Smoggy Word Clouds for Symmetry

These word clouds were found on the nebulous internet. The origin of word clouds can be traced back to the 1970. Their popularization has raised with computer programs. In a word cloud, in the first place (size or/and center) we have the word which is the object of study, then, surrounding it, we have other words.

There are no precise rules for choosing these words, the basic idea is that they are *related* in some way to the keyword, or to use another word of philosophical significance, *associated* to it. A cloud can be drawn using some laws for these relations such as the laws governing a *synset* in the project *WordNet* developed by Princeton University. These are conceptual relationships, which include hyponyms, hypernyms, sister terms, derivationally related forms. Language is important, but considering what has been said in the previous section, it is important to take in account also thought and reality. We can use the terminology *ConceptNet* or, even better, *NotionNet*. The two word clouds of Fig. 7 are more or less satisfactory. Let us construct a better one.

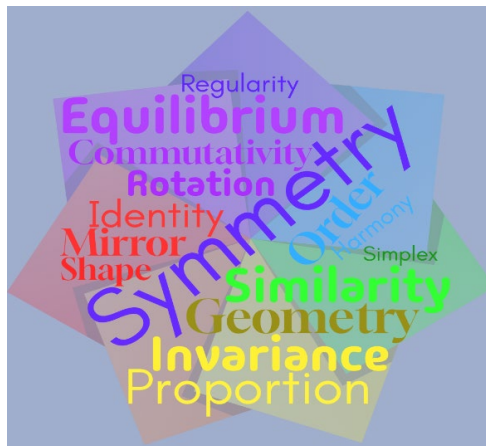


Fig.8 A Pure Word Cloud for Symmetry

Let us first explain what we do not have in this cloud. Firstly, we do not have proper names such as Herman Weyl, Dénes Nagy, or Ioannis Vandoulakis. Secondly, we do not have words of the same family such as asymmetry, dissymmetry, or antisymmetry. Thirdly, we do not have opposed ideas such as chaos, confusion, or disorder. Fourthly, we do not have very general notions such as concept, relation, property. Fifthly, we do not have examples such as butterfly, square, or snowflake. What is therefore left? Qualities, features, aspects which characterize or/and help to define symmetry.

To design a word cloud for symmetry for an umbrella notion like symmetry is a real challenge!

3 HEXAGON OF SYMMETRY

An important way to answer a question of type “What is X?” is a *negative* approach. But this approach must be meaningful, to say that symmetry is not fear is not very useful, although one may reflect about the relation between symmetry and emotions.

A good way to proceed is to use the theory of oppositions. Let us have a look at the following hexagon of opposition for symmetry:

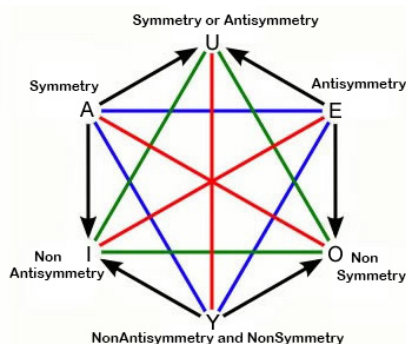


Fig. 9 The Negative Hexagon of Symmetry

This hexagon is based on two notions which are *contrary*, symmetry and antisymmetry. It means that something cannot be at the same symmetrical and antisymmetrical. Antisymmetry is the reverse of symmetry in the sense that in case aRb , necessarily it is not the case that bRa . A non-antisymmetrical relation is a relation that is not antisymmetric but can be symmetric and a relation which is non-symmetrical is a relation that is not symmetrical but can be antisymmetrical. The position Y is the case where the relation is both non-symmetrical and non-antisymmetrical, which are two *subcontrary* notions, excluding the case of symmetry and antisymmetry, and it is *contradictorily* opposed to a relation which is either symmetric or antisymmetric (U corner).

The words “asymmetry” and “dissymmetry” exist and are used sometimes are synonymous, their meaning being floating. A reasonable way to fix it is to choose to

identify asymmetry with the 0-corner and dissymmetry with the Y-corner. It is normative but to go out of confusion normativity is useful. To answer question “What is X?” in a descriptive way either enumerating all the things falling under the X notion, or giving all the meanings attached to the name of such notion is not a satisfactory procedure, which was criticize by Plato. We have then the following hexagon:

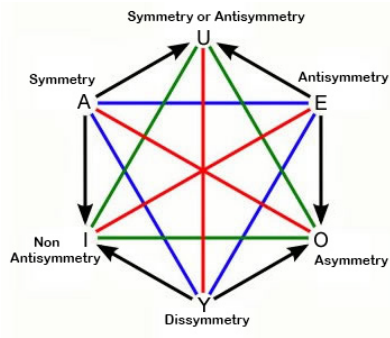


Fig. 10 The Positive Hexagon of Symmetry

An interesting point is to try to give flesh to this skeleton. Where to locate human body? It is in fact dissymmetric, using this hexagonal definition, as we have pointed out in a previous paper and as it can be seen on the following picture:

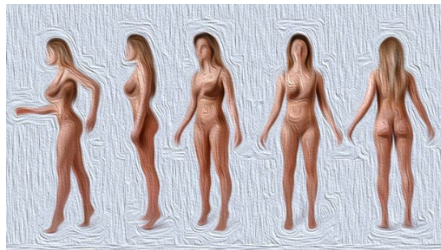


Fig .11 Dissymetry of Female Body

Not only the woman body is dissymmetric, but also the man body:



Fig .12 Dissymetry of Male Body

However, man and woman can be considered as forming a harmonious *yin yang* symmetry:



Fig .13 Yin Yang Symmetry of Female and Male

What about a storm? In principle it is something which is chaotic, there is not regularity, but is it antisymmetric, asymmetric, or dissymmetric?



Fig .14 Storm in the Universe of Symmetry

If we consider that *Cosmos*, ruled by order, is the negation of *Chaos*, *Chaos* is not asymmetric because order does not reduce to symmetry, funny enough a relation of strict order is by definition antisymmetric. For this reason, it would also a bit absurd to identify *Chaos* with antisymmetry. Non-antisymmetry would also not work because it does not exclude symmetry. At the end, it seems that the best place for *Chaos* is the hexagon of symmetry is dissymmetry, side by side with human body.

(IN NON LIEU OF) CONCLUSION

Symmetry is a fascinating notion and we have tried to catch it through three different types of methodologies. They are shedding complementary lights. It does not mean that we have now reached the essence of symmetry, if any, but we have at least a better idea of what it is and what it is not, where it is located, and how to use this umbrella in case of storm.

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